# MATHEMATICAL AND OPTIMIZATION ANALYSIS OF A MINIATURE STIRLING CRYOCOOLER

Thesis submitted in partial fulfillment of the requirements for the degree of

**Bachelor of Technology (B. Tech)**

**In**

**Mechanical Engineering**

By


Yatin Chhabra (107ME025)

Devaraj. V (107ME015)


Under the guidance of

Prof. R.K. Sahoo


**NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA**

## Certificate of Approval

This is to certify that the thesis entitled **Mathematical and Optimization Analysis of a Miniature Stirling Cryo-cooler** submitted by ***Mr. Yatin Chhabra*** and ***Mr. Devaraj. V*** has been carried out under my supervision in partial fulfillment of the requirements for the Degree of **Bachelors of Technology** (**B. Tech**) in Mechanical Engineering at National Institute of Technology Rourkela, and this work has not been submitted elsewhere before for any other academic degree/diploma.

.................................................

Dr. R. K. Sahoo
Professor
Department of Mechanical Engineering
National Institute of Technology, Rourkela

## Abstract

In the given report, a comprehensive analytical model of the working of a miniature Stirling Cyo-cooler is presented. The motivation of the study is to determine the optimum geometrical parameters of a cryo-cooler such as compressor length, regenerator diameter, expander diameter, and expander stroke. In the first part of the study, an ideal analysis is carried out using the Stirling Cycle and basic thermodynamics equations. Using these equations, rough geometrical parameters are found out.

In the second part of the study, a more comprehensive Schmidt's analysis is carried out. In this analysis, pressure and volume variations are considered sinusoidal and based on these, various equations regarding efficiency and COP are derived. Various graphs are generated in MATLAB plotting Refrigeration and Work done w.r.t to various geometrical parameters. With the help of these graphs, the net refrigeration obtained is calculated for a given geometry of cryo-cooler.. This model provides a more accurate picture of the cryo-cooler. However in this analysis, regenerator efficiency is considered 100 % which is not true in practical cases.

In the third and final part of the study, optimization of regenerator is carried out. This part is based on Ackermann's analysis in which various looses taking place inside a regenerator are considered and accounted for. These looses are minimized using an iterative cycle and optimum regenerator dimensions are obtained. Thus the geometrical results obtained from the third part of the study are expected to be most accurate as it accounts for most of the looses taking place inside a cryo-cooler.

## Acknowledgment

I was privileged to have worked under the supervision of Prof. R.K. Sahoo. His invaluable advise, enthusiasm and support throughout my project provided me with the motivation to stay on track and fulfill my potential.

I would also like to thank Prof. S.K. Sarangi for his timely advise and encouragement which helped me get through the difficult periods during the project.

In the end I would also like to extend my gratitude to the current Masters Students of Mechanical Engineering department for their help regarding the use of computational software's like FLUENT and MATLAB.

# Table of Contents

1) INTRODUCTION – HISTORICAL REVIEW………………………………1
2) STIRLING CYCLE……………………………………………………….2
3) IDEAL ANALYSIS OF MINIATURE STIRLING CRYOCOOLER………4
4) SCHMIDT CYCLE……………………………………………………..7
5) SIMULATION WITH SCHMIDT CYCLE…………………………………9
6) RESULTS AND DISCUSSIONS……………………………………..10
7) SHORTCOMINGS OF SCHMIDT ANALYSIS…………………………..20
8) SCOPE FOR FUTURE WORK…………………………………………….21
9) REGENERATOR OPTIMIZATION……………………………………..22
10) OPTIMIZATION ANALYSIS……………………………………………23
11) OPTIMIZATION PROCEDURE……………………………………..29
12) GRAPHS ………………………………………………………………34
13) CONCLUSION…………………………………………………………42
14) REFERENCES…………………………………………………………43
15) APPENDIX……………………………………………………….…44

# List of Figures


1) Figure 1: Ideal Stirling Cycle
2) Figure 2: Pressure vs Total Volume
3) Figure 3: Pressure vs Expansion and Compression Volume
4) Figure 4: $P_{max}$ vs $T_c$
5) Figure 5: Qe vs $T_C$
6) Figure 6: W vs $T_C$
7) Figure 7: COP vs $T_C$
8) Figure 8: Pmax vs alpha
9) Figure 9: W vs alpha
10) Figure 10: COP vs alpha
11) Figure 11: Qe vs alpha
12) Figure 12: Pmin vs y (VD/VE)
13) Figure 13: Qe vs y (VD/VE)
14) Figure 14: W vs y (VD/VE)
15) Figure 15:COP vs y (VD/VE)
16) Figure 16: Pmax vs x (VC/VE)
17) Figure 17: COP vs x (VC/VE)
18) Figure 18: W vs x (VC/VE)
19) Figure 19: Qe vs x (VC/VE)
20) Figure 20: Regenerator Porosity vs $Q_c$ (Watts)
21) Figure 21: Regenerator Porosity vs Qreg (Watts)
22) Figure 22: Regenerator Porosity vs Wpf (Watts)
23) Figure 23: Regenerator Porosity vs Wprv (Watts)
24) Figure 24: Frequency vs Qc
25) Figure 25: Frequency vs Qreg
26) Figure 26: Frequency vs Qpf
27) Figure 27: Frequency vs Qprv
28) Figure 28: Tc vs Qc
29) Figure 29: $T_{cold}$ vs Qreg
30) Figure 30: $T_{cold}$ vs Qpf
31) Figure 31: $T_{cold}$ vs Qprv
32) Figure 32: Volume Ratio (Vc/Ve) vs Qc
33) Figure 33: Volume Ratio (Vc/Ve) vs Qreg
34) Figure 34: Volume Ratio (Vc/Ve) vs Qpf
35) Figure 35: Volume Ratio (Vc/Ve) vs Qprv

# List of Tables:

1) Table 1: : Stirling Cycle Refrigerator Design Parameters
2) Table 2: First Estimate Of Losses in a Stirling Cryocooler
3) Table 3: Calculated Regenerator Parameters from the Optimization Equations
4) Table 4: Calculated Regenerator Heat Transfer Parameters from Optimized Regenerator Values
5) Table 5: Calculated Regenerator Losses

# NOMENCLATURE:

1) $A_{ff}$ = Fluid axial free flow area
2) $A_m$ = Matrix thermal conduction heat transfer area
3) $A_r$ = Total regenerator frontal area
4) $A_s$ = Matrix total heat transfer area
5) $C_c$ = Cold fluid heat capacity rate
6) $C_h$ = Warm fluid heat capacity rate
7) $C_m$ = Matrix heat capacity rate
8) $C_{max}$ = The larger of Cc and Ch
9) $C_{min}$ = The smaller of Cc and Ch\
10) $C_p$ = Specific heat at constant pressure
11) $C_v$ = Specific heat at constant volume
12) $D_d$ = Displacer diameter
13) Dh = Hydraulic diameter ($D_h = 4*r_h$)
14) $D_r$ = Regenerator diameter
15) d = Screen wire diameter
16) f = Fluid coefficient of friction
17) fr = Frequency
18) G = Mass flow rate per unit free flow area
19) h = Heat transfer coefficient
20) Ie = Regenerator thermal inefficiency
21) $K_f$ = Fluid thermal conductivity
22) $K_m$ = Matrix thermal conductivity
23) L = Regenerator length
24) M = Molecular weight
25) $M_f$ = Mass of the fluid
26) $M_m$ = Mass of the matrix material
27) m' = Mass flow rate
28) $P_{max}$ = Maximum cycle pressure
29) $P_{min}$ = Minimum cycle pressure

30) Ps = System pressure

31) Pa = Pressure ratio

32) $p_{so}$ = System pressure amplitude

33) $Q_c$ = Heat conduction

34) $Q_h$ = Heat transferred between fluid and matrix material

35) $Q_{net}$ = Net cryocooler refrigeration

36) $Q_{reg}$ = Regenerator thermal losses

37) $T_a$ = Ambient room temperature

38) $T_{cold}$ = Cold end temperature

39) $T_w$ = Warm end temperature

40) t = Time

41) $V_e$ = Expansion space volume

42) $V_{cs}$ = Compression space volume

43) $V_m$ = Volume occupies by the matrix material

44) $V_r$ = Regenerator total volume

45) $V_{rv}$ = Regenerator void volume

46) $v_{eo}$ = Expansion space volume amplitude

47) $W_{pv}$ = Gross mechanical refrigeration produced

48) W = Mechanical power

49) $X_d$ = Displacer position

50) $X_p$ = Compressor piston position

51) S = Expander stroke

52) $P_{mean}$ = Mean cycle pressure

53) p = instantaneous cycle pressure

54) $P_{max}$ = maximum cycle-pressure

55) $P_{min}$ = Minimum cycle-pressure

56) $P_{mean\,=}$ Mean cycle-pressure

57) W = Work Done by compressor

58) $Q_e$ = heat transferred to the working fluid in expansion space

59) $Q_c$ = heat transferred in the compression space

60) COP = coefficient of performance of the cryocooler

61) R = characteristic gas constant of the working fluid

62) $T_C$ = Temperature of the working fluid in the compression space

63) $T_D$ = Temperature of the working fluid in the dead space

64) $T_E$ = Temperature of the working fluid in expansion space

65) $V_C$ = swept volume of the compression space

66) $V_E$ = swept volume of the expansion space

67) $V_D$ = swept volume of the dead space

68) $x = V_C/V_E$, swept volume ratio

69) $y = V_D/V_E$, dead volume ratio

70) $t = T_C/T_E$, temperature ratio

71) $V_T = (V_E + V_C + V_D)$

72) $\alpha$ = angle by which volume variations in expansion space lead those in the compression space

73) A = a factor $[t^2 +2*t*x*\cos(\alpha)+x^2]^{0.5}$

74) $S = (V_D*T_C)/(V_E*T_D)$

75) B = a factor (t+x+2*S)

76) $\delta = A/B = [\ t^2 + x^2 + 2*t*x*\cos(\alpha)\ ]^{0.5}/(t+x+2*S)$

77) $\theta = \mathrm{atan}[\{x*\sin(\alpha)\}]/(t+k+2*S)$

78) $\varphi$= crank angle

# INTRODUCTION

## HISTORICAL REVIEW

A Stirling engine operates in a closed thermodynamic regenerative cycle with the same working fluid repeatedly compressed and expanded at different temperature levels so there is a net conversion of heat to work or vice versa. It can be used as a cooling engine, prime mover, heat pump, or pressure generator.

As a cooling machine, it extends back as far as 1817. In 1834 John Herschel conceived the closed cycle regenerative cooling engine for making ice. The concept was not reduced for practice for 30 years.

The first Stirling cooling engines were made by Alexander Kirk, a Scottish Engineer working at oil works in Scotland. He constructed cooling engines for a variety of application both in Great Britain and oversees. However the Kirk machines were never made in large numbers or used to achieve cryogenic temperature.

The next significant development occurred with the start of the Phillips cooling engine research about 1946. It followed a decade of effort already invested in small Stirling engine prime movers. The Phillips was the first time that substantial effort was invested simultaneously in Stirling engines for power and cooling application.

Military interest in infrared thermal imaging equipment for night vision and heat seeking missile guidance focused research attention on miniature cryocoolers in the 1960s. This is now sustained by continued military interests, by spacecraft instrument applications, and by incipient applications of superconducting electronic and electrical systems on a broad front.

The objective of this work is to perform mathematical analysis to design a miniature Stirling cryocooler capable of giving 250 mW of cooling capacity at a temperature of 80 K.

# STIRLING CYCLE

The **Stirling cycle** is a thermodynamic cycle that describes the general class of Stirling devices. This includes the original Stirling engine that was invented, developed and patented in 1816 by Reverend Dr. Robert Stirling with help from his brother, an engineer. [2]

The cycle is reversible, meaning that if supplied with mechanical power, it can function as a heat pump for heating or refrigeration cooling, and even for cryogenic cooling. The cycle is defined as a closed-cycle regenerative cycle with a gaseous working fluid. "Closed-cycle" means the working fluid is permanently contained within the thermodynamic system. This also categorizes the engine device as an external heat engine. "Regenerative" refers to the use of an internal heat exchanger called a regenerator which increases the device's thermal efficiency. [2]

The cycle is the same as most other heat cycles in that there are four main processes: 1.Compression, 2. Heat-addition, 3. Expansion and 4. Heat removal. However, these processes are not discrete, but rather the transitions overlap.

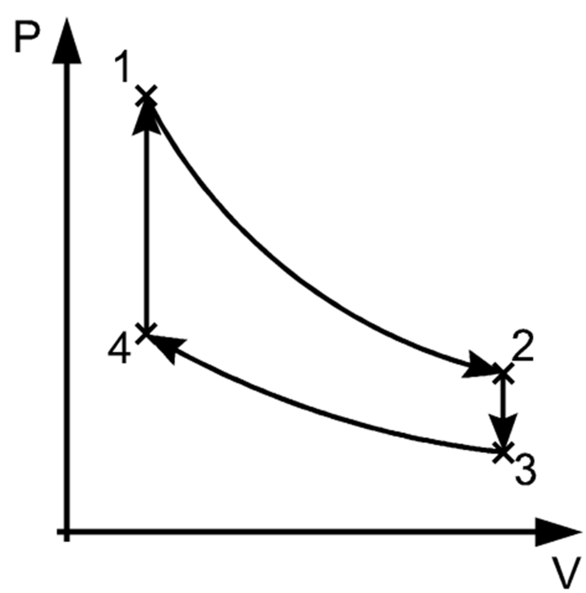



Fig. 1: Ideal Stirling cycle [1]

The **idealized** Stirling cycle consists of four thermodynamic processes acting on the working fluid:

- Points 1 to 2, Isothermal Expansion. The expansion-space is heated externally, and the gas undergoes near-isothermal expansion.
- Points 2 to 3, Constant-Volume (known as iso-volumetric or isochoric) heat-removal. The gas is passed through the regenerator, thus cooling the gas, and transferring heat to the regenerator for use in the next cycle.
- Points 3 to 4, Isothermal Compression. The compression space is intercooled, so the gas undergoes near-isothermal compression.
- Points 4 to 1, Constant-Volume (known as iso-volumetric or isochoric) heat-addition. The compressed air flows back through the regenerator and picks-up heat on the way to the heated expansion space.

# IDEAL ANALYSIS

In the following section we present an ideal analysis of the miniature Stirling cryocooler. We assume the maximum and minimum pressure in the cryocooler to be 35 and 4 bar respectively and the speed of the machine to be 50 Hz. Further assuming ideal gas behavior and applying ideal gas equations we find out the approximate dimensions of the compression chamber.

Nomenclature:

a) $T_C$ = Temperature in the compression chamber
b) $T_E$ = Temperature in the expansion chamber
c) m = Mass flow rate of the helium in Kg/s
d) R = Characteristic gas constant
e) r = Ratio of maximum volume $V_1$ to minimum volume $V_2$

Initial Values:

a) Maximum pressure = 35 bar
b) Minimum pressure = 4 bar
c) Running Speed = 50 Hz

1) <u>Isothermal compression process (2-1)</u>

   $T_2 = T_1 = T_C = 320$ K

   $P_1 * V_1 = P_2 * V_2$

   $r = V_1/V_2 = P_2/P_1 = 35/P_1$

2) <u>Constant volume regenerative cooling (1-4)</u>

   $V_2 = V_3$

   $P_2/T_2 = P_3/T_3$

   $P_3 = P_2 * T_3/T_2 = 35 * 80/320 = 8.75$ bar

3) <u>Isothermal expansion process (4-3)</u>

$T_3 = T_4 = T_E = 80$ K

$P_3 * V_3 = P_4 * V_4$

$P_3 / P_4 = V_4 / V_3 = V_1 / V_2 = r$

r = 8.75/4 = 2.19.

4) <u>Constant volume regenerative heat transfer (3-2)</u>

$V_4 = V_1$

$P_4/T_4 = P_1/T_1$

$P_1 = 4 * 320/80 = 16$ bar

Let rate of heat transferred to expansion space = $W_2$

Let rate of heat addition to compression chamber = $W_1$

$W_2 = m * R * T_E * \ln(1/r)$

$= m * 2.0785 * 80 * \ln(1/2.19)$

$W_1 = m * 2.0785 * T_C * \ln(1/r)$

$= m * 2.0785 * 320 * \ln(1/2.19)$

Now we know that the rate of heat transferred to expansion space = 0.250 W

Therefore, $W_2 = 0.250$ W

Solving we get **m = <u>$1.91 * 10^{-3}$ Kg/s</u>**

Assuming ideal gas behavior,

$P_1 * V_1 = m * R * T_1$

Thus, **$V_1$ = <u>$798.1 * 10^{-6}$ $m^3/s$</u>**

For running speed = 50 rps,

Actual volume in cylinder

$V = 798.1 * 10^{-6}/50$

⇨ **V = <u>$15.962 * 10^{-6}$ $m^3$</u>**

## Dimensions of compressor cylinder

In compressor cylinder,

$V = \pi *( D^2/4) * (L+c)$

D = Cylinder diameter

L = Stroke length

c = Clearance length

$r = V_1/V_2 = 2.19$

$(L+c)/c = 2.19$

$L/c = 1.19$

$c = L/1.19$

Now assuming, D/L =2

$V = [(pi * L^3)]*(2.19/1.19) = 15.962 *10^{-6}\ m^3$

**L = 1.40 cm**

**D = 2.80 cm**

# SCHMIDT CYCLE

The classical analysis of the operation of Stirling engines is due to Schmidt (1861). The theory provides for the harmonic motion of the reciprocating elements, but retains the major assumption of isothermal compression and expansion and of perfect regeneration. It, thus remains highly idealized, but is certainly more realistic than the ideal Stirling Cycle. [3]

Assumptions of the Schmidt cycle:

1) The regenerative process is perfect.
2) The instantaneous pressure is same throughout the system.
3) The working fluid obeys the characteristic gas equation, PV=RT.
4) There is no leakage, and the mass of the working fluid remains constant.
5) The volume variations in the working space occur simultaneously.
6) There are no temperature gradients in the heat-exchangers.
7) The cylinder wall, and piston, temperatures are constant.
8) There is perfect mixing of cylinder contents.
9) The temperature of the working fluid in ancillary spaces is constant.
10) The speed of the machine is constant.
11) Steady state conditions are established.

**Basic Equations: [3]**

Volume of expansion space:

$$\mathbf{Ve = \tfrac{1}{2} * V_{E*} [1 + \cos(\varphi)]}$$

Volume of compression space:

$$\mathbf{Vc = \tfrac{1}{2} * V_C * [1 + \cos(\varphi - \alpha)]}$$

$$\mathbf{= \tfrac{1}{2} * x * V_E * [1 + \cos(\varphi - \alpha)]}$$

Using the above equations and performing Schmidt's calculations we get the following equations:

Instantaneous Pressure,

$$p = P_{max} *(1-\delta)/ [1 +\delta*\cos(\varphi - \theta)]$$

Or,

$$p = P_{min} *(1+\delta)/ [1 + \delta*\cos(\varphi - \theta)]$$

Therefore, $P_{max} / P_{min} = (1+\delta)/ (1 - \delta)$

Mean Cycle-Pressure,

$$P_{mean} = P_{max} *[(1-\delta)/ (1+\delta)]^{0.5}$$

So,

$$P_{max} = P_{mean} *[(1+\delta)/(1-\delta)]^{0.5}$$

And,

$$P_{min} = P_{mean} *[(1-\delta)/(1+\delta)]^{0.5}$$

Heat Transferred in expansion and compression space,

$$Q_e = [\pi * P_{mean} * V_E * (\delta) * \sin(\theta) ]/ [ 1+ (1 - \delta^2)^{1/2} ]$$

$$Q_c = [ \pi * P_{mean} * V_E * x * (\delta) * \sin( \theta-\alpha ) ] / [ 1 + (1 - \delta^2 )^{1/2} ]$$

Work done in compressor,

$$W = Q_e - Q_c$$

Coefficient of performance,

$$COP = Q_e/(Q_e-Q_c)$$

# SIMULATION WITH SCHMIDT CYCLE

A number of simple programs written in MATLAB are presented in this section. The programs are used to obtain different kinds of plots by varying different parameters present in the equations above.  The programs are given in Appendix at the end of the report.

The plots which we obtain enable us to find the optimum parameters which are necessary to obtain the required refrigeration capacity of miniature Stirling cryocooler.

# RESULTS AND DISCUSSION

Using the matlab codes given, graphs of pressure vs expansion volume, compression volume and total volume, and $P_{max}$, $P_{min}$, Qe, W, COP vs TC, α, x, y were plotted which are shown below.

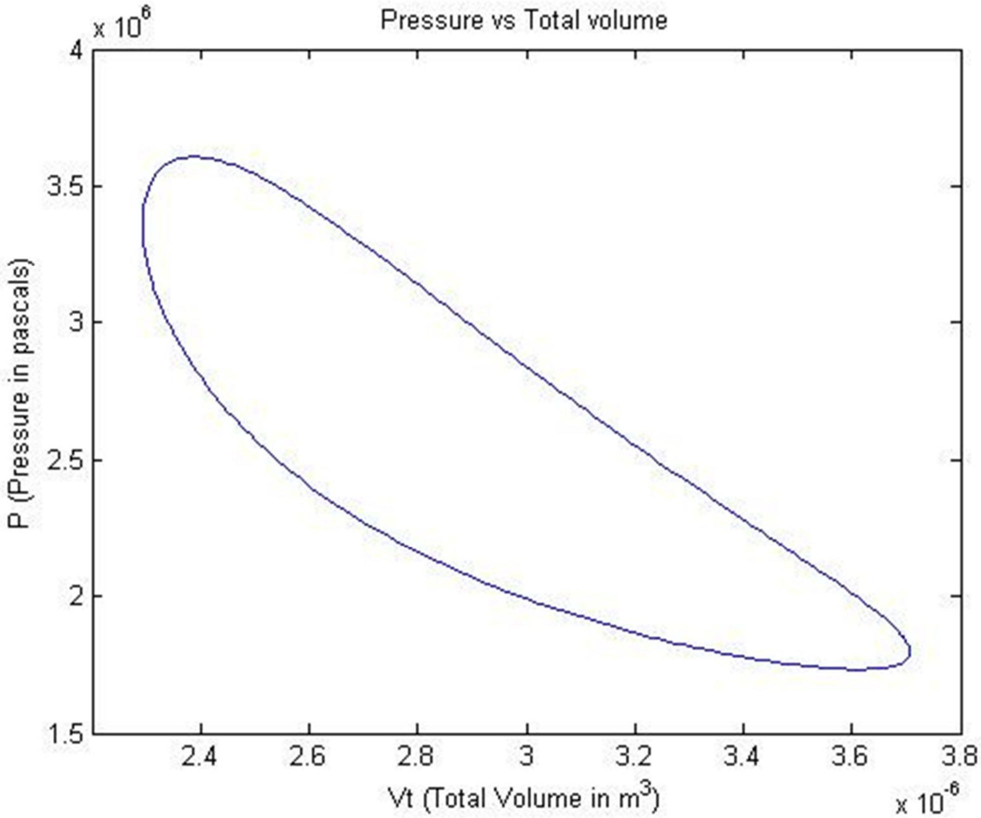



Fig. 2: Pressure vs Total Volume

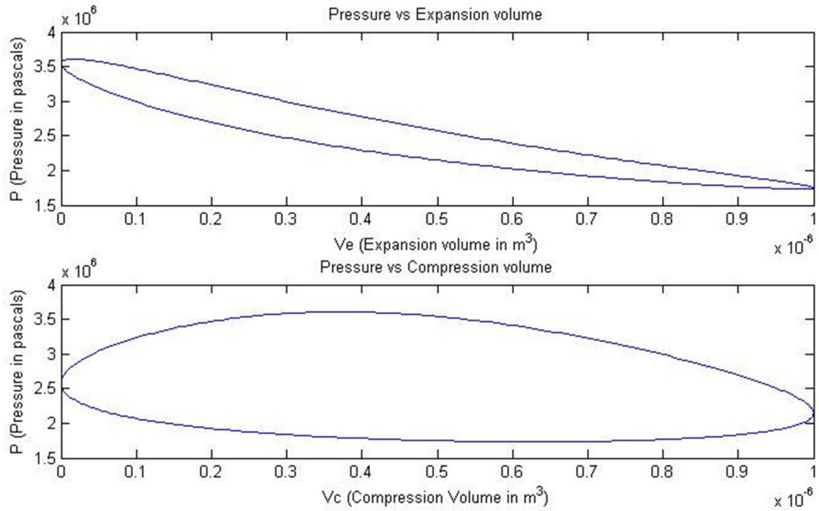



Fig. 3: Pressure vs Expansion and Compression Volume

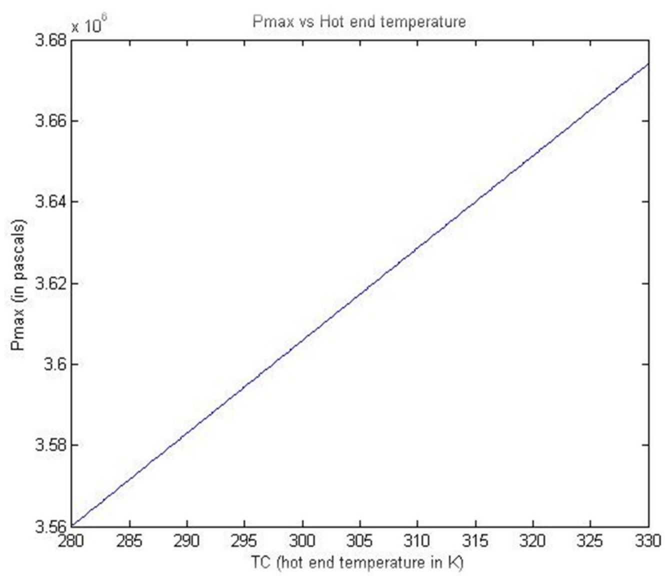



Fig. 4: $P_{max}$ vs $T_c$

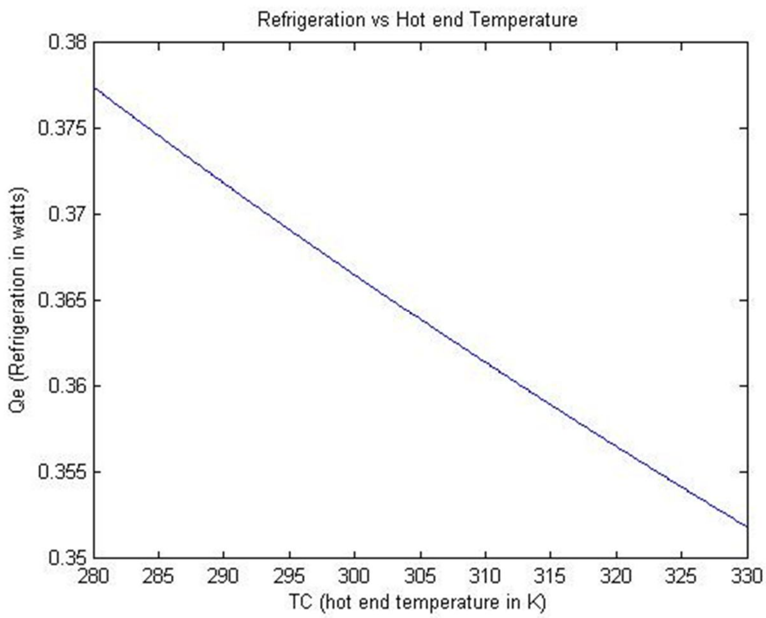



Fig. 5: Qe vs TC

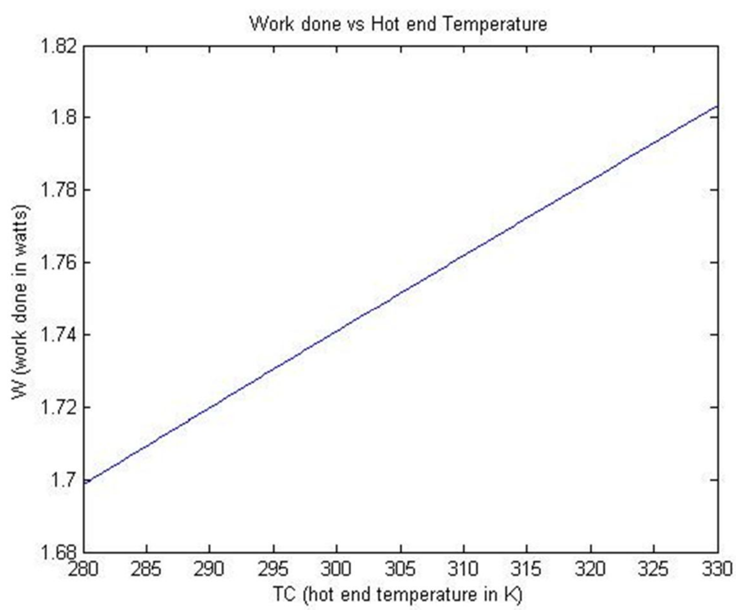



Fig.6: W vs TC

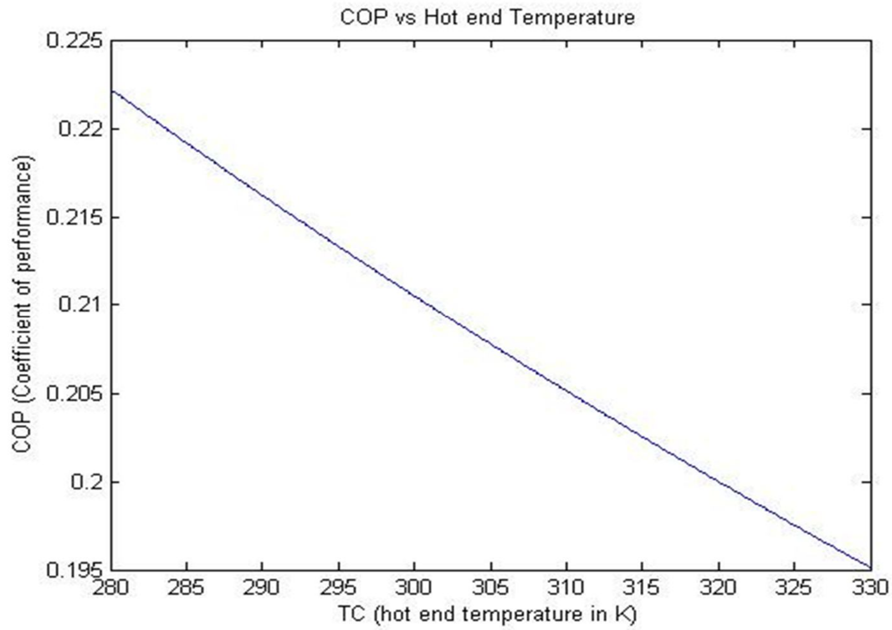



Fig.7: COP vs TC

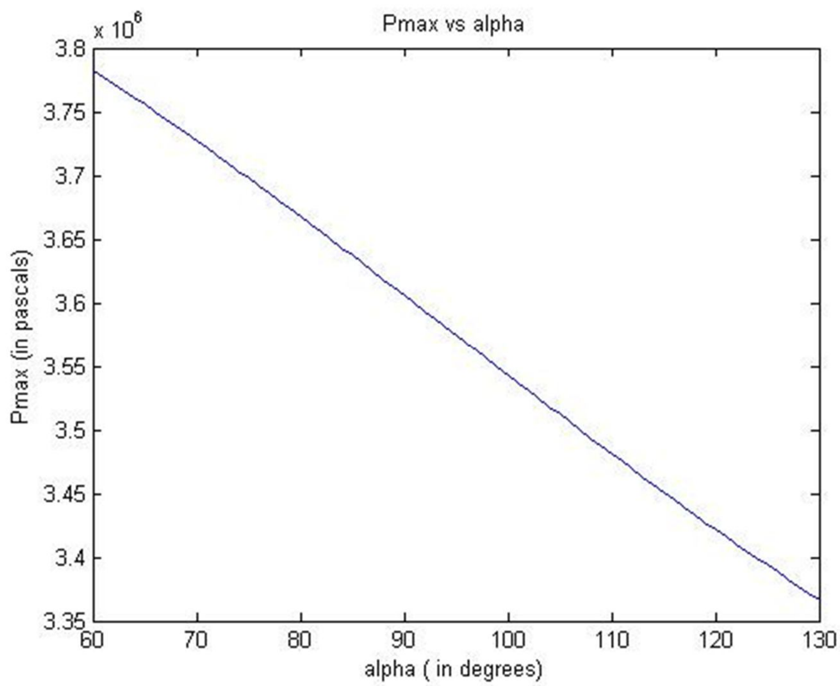



Fig.8: Pmax vs alpha

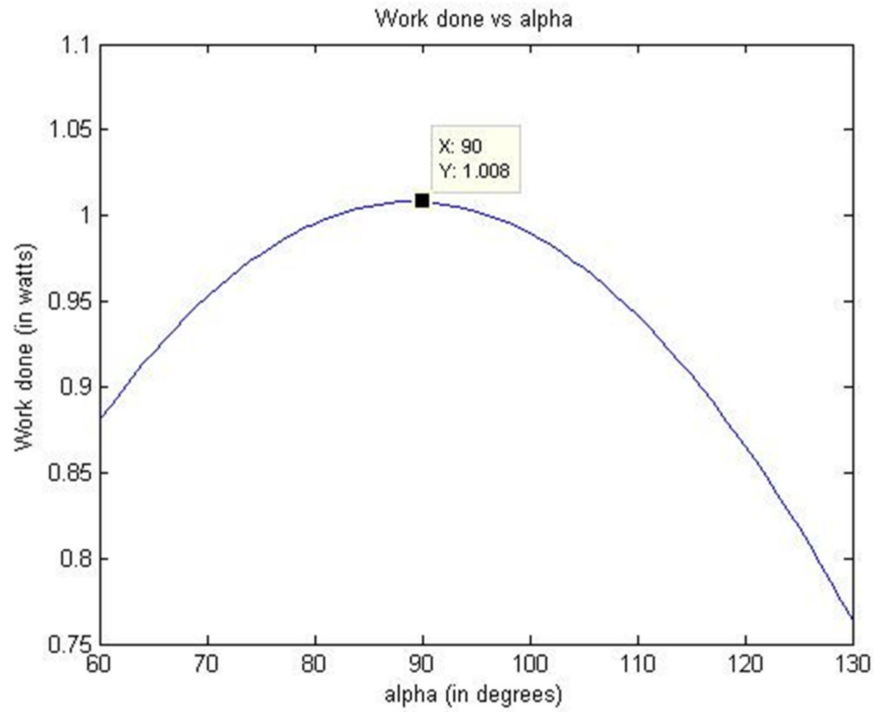



Fig. 9: W vs alpha

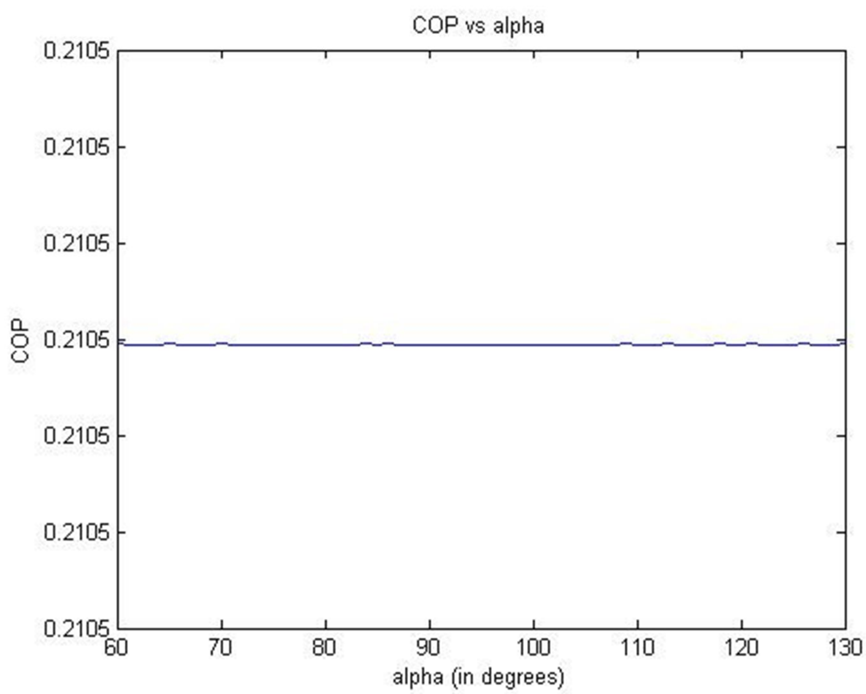



Fig. 10: COP vs alpha

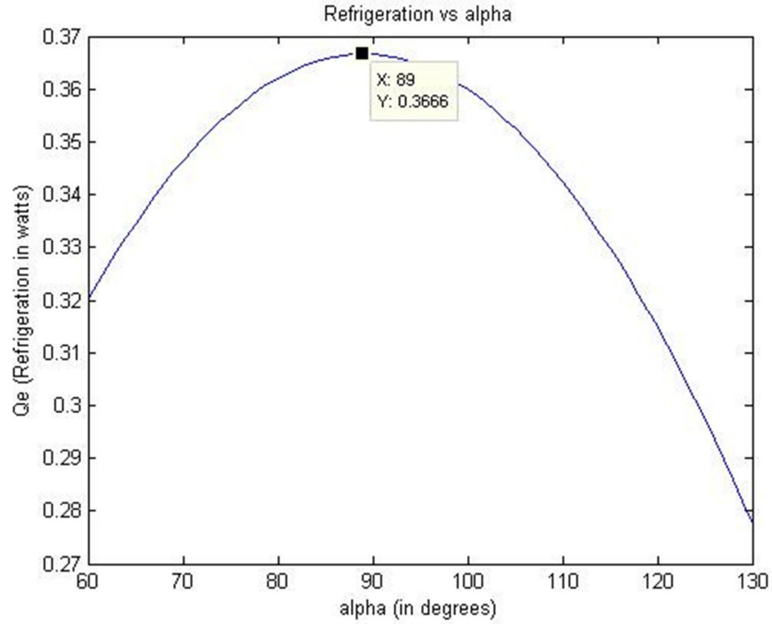



Fig. 11: Qe vs alpha

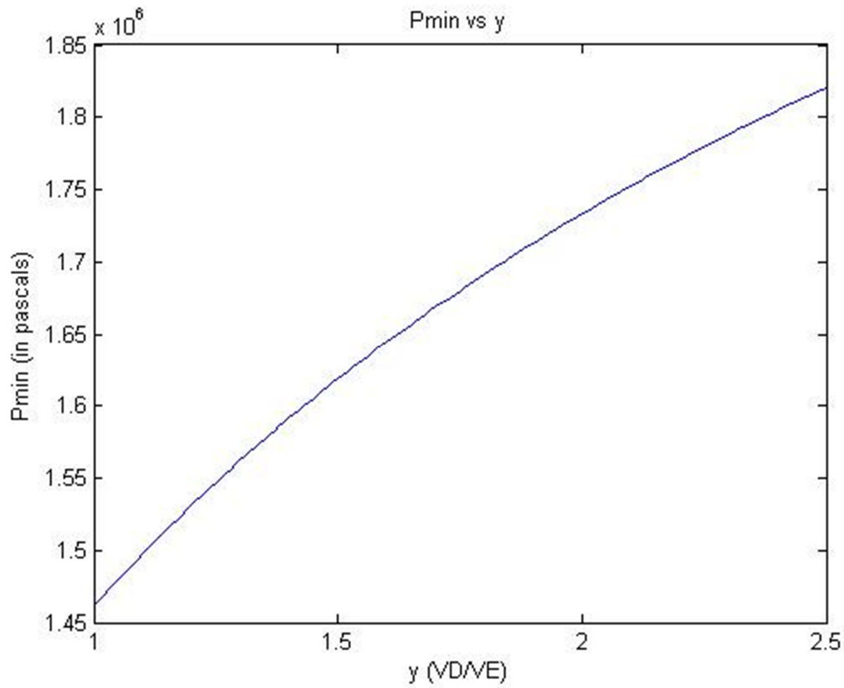



Fig. 12: Pmin vs y (VD/VE)

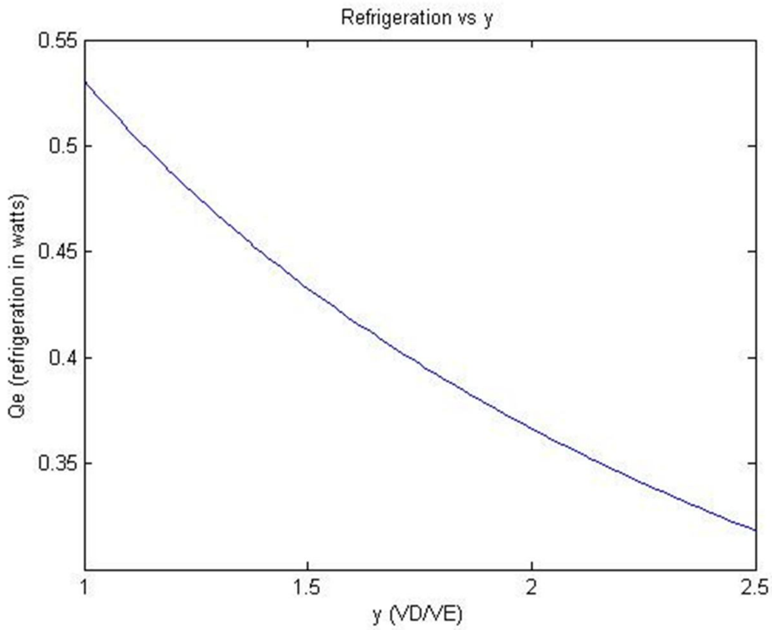



Fig. 13: Qe vs y (VD/VE)

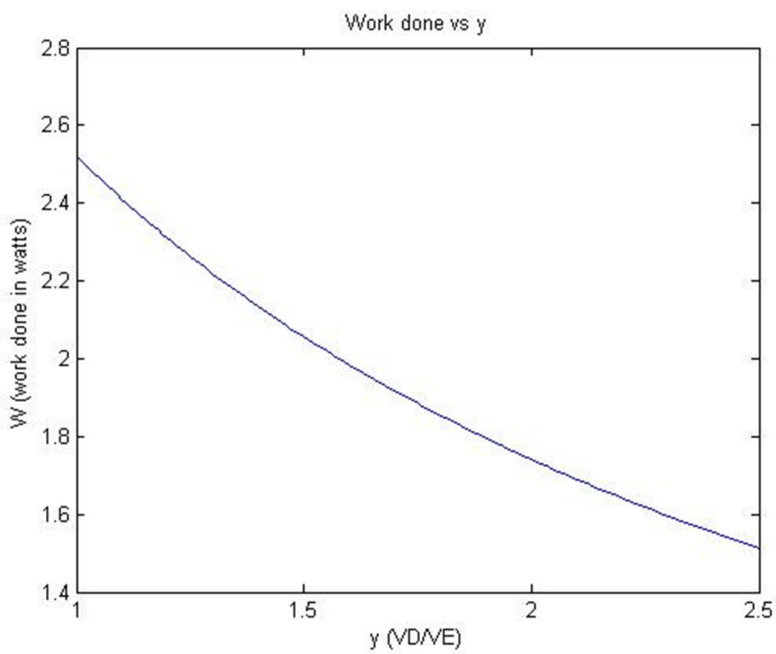



Fig. 14: W vs y (VD/VE)

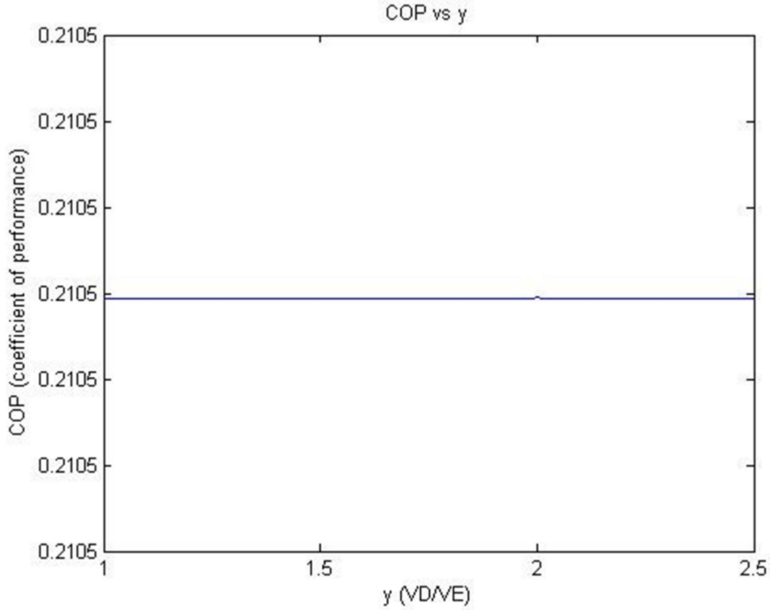



Fig. 15: COP vs y(VD/VE)

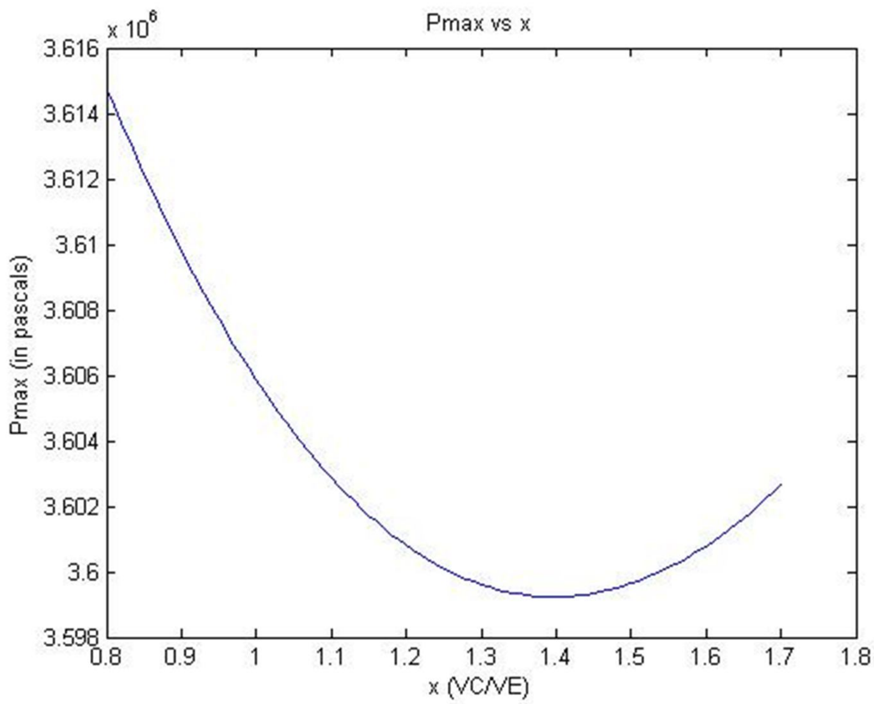



Fig. 16: Pmax vs x (VC/VE)

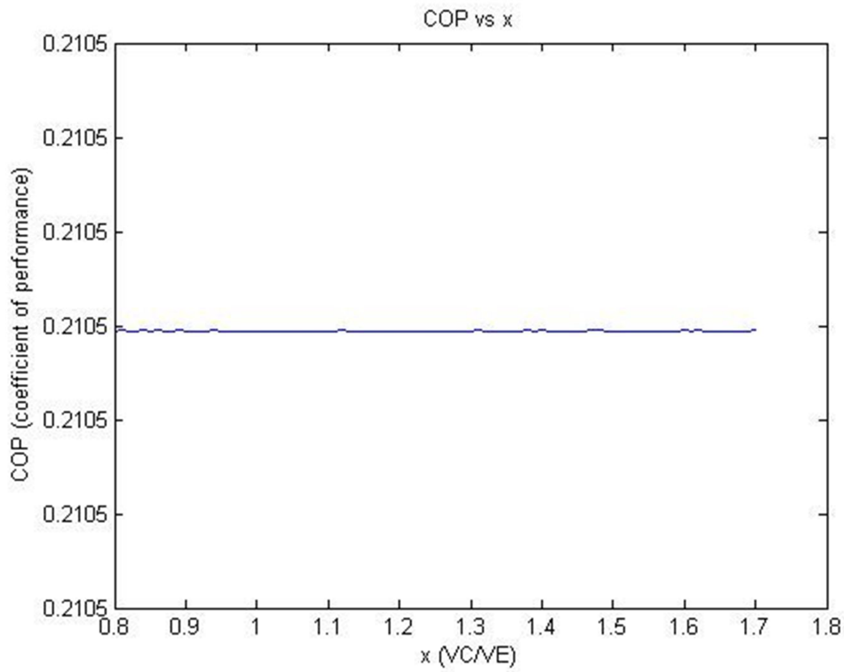



Fig. 17: COP vs x (VC/VE)

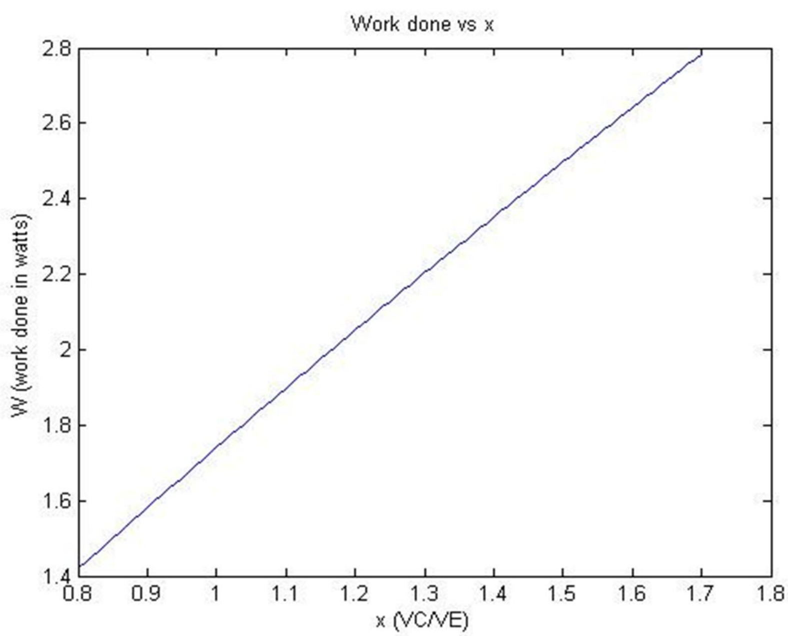



Fig. 18: W vs x(VC/VE)

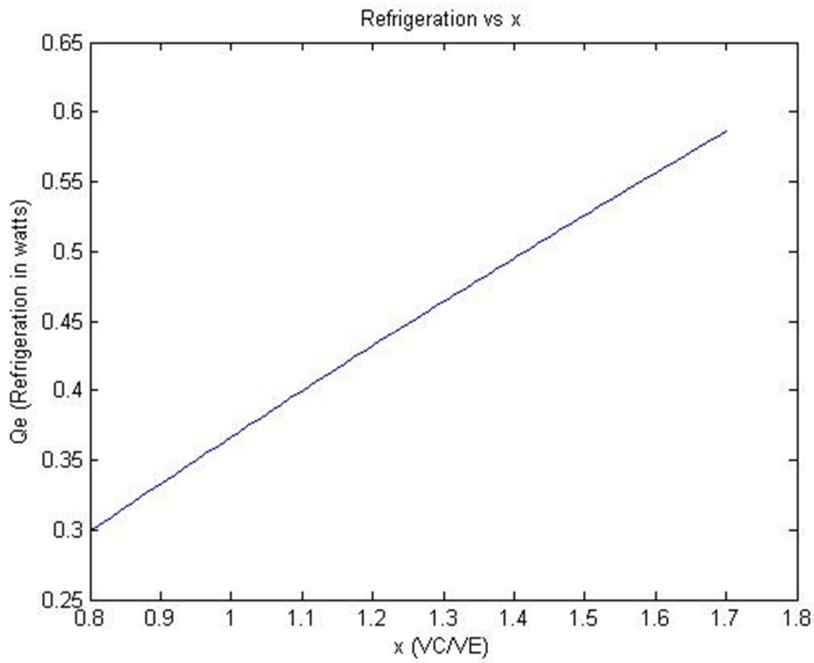



Fig. 19: Qe vs x (VC/VE)

From the graphs above following observations are made,

1) To achieve maximum refrigeration capacity, x must be as high as possible (restricted by $P_{max}$), α must be taken 90 degrees, y must be as low as possible (restricted by $P_{max}$) and $T_C$ must be as low as possible in a given working environment.
2) For minimum $P_{max}$, x must be 1.4, α must be as high as possible (restricted by refrigeration capacity), y must be as high as possible (restricted by refrigeration capacity) and $T_C$ must be as low as possible in a given working environment.

So, refrigeration capacity achieved and $P_{max}$ for optimum parameters,

x = 1.4,

y = 2,

α = 90,

$T_C$ = 300,

From Schmidt's calculations,

**Qe = 495 mW**

**Pmax = 36 bar.**

# SHORTCOMINGS OF SCHMIDT ANALYSIS

One of the biggest shortcomings of Schmidt cycle is the assumption that the efficiency of the regenerator is 100%. In reality the efficiency of the regenerator is expected to be around 98%. This drop in regenerator efficiency will reduce the value of $P_{max}$ and increase the value of $P_{min}$ thereby reducing pressure ratio and the refrigeration capacity will be reduced substantially(from 495 to around 250).

Also the pressure losses in the regenerator due to large flow resistance of meshes is neglected which in reality can have a substantial impact on the performance of a Stirling cryocooler.

# SCOPE FOR FUTURE WORK

The future work will consist of incorporating the regenerator efficiency into the given Schmidt model; by doing so we will be able to obtain more accurate parameters to get the required refrigeration capacity. Taking regenerator effectiveness into account is expected to bring the changes in following quantities:

1) Refrigeration capacity – The refrigeration capacity is expected to decrease. This is due to the fact that the heat transfer between the fluid and the regenerator is not 100% and hence the temperature of fluid reaching the expansion or the compression chamber is slightly more or less than the ideal values we assumed in the previous sections. Hence the actual refrigeration capacity of this cryocooler is expected to be around 250 mW instead of 495 mW.
2) Maximum pressure ($P_{max}$) -The value of maximum pressure is expected to decrease.
3) Minimum pressure ($P_{min}$) – The value of minimum pressure is expected to rise.
4) COP – Since the refrigeration capacity is reduced hence, the COP is reduced accordingly.
5) Power input – In order to increase the reduced refrigeration capacity, the power input has to be increased.
6) Value of x – From the graph obtained between $P_{max}$ and x in the Schmidt analysis above, we observe that as the value of $P_{max}$ decreases, the value of x increases correspondingly. Hence by incorporating the regenerator efficiency we can expect the value of x (i.e. $V_C$) to increase with corresponding decrease of $P_{max}$.

# REGENERATOR OPTIMIZATION

Regenerator optimization is the process of choosing regenerator design parameters that maximize system performance. For cryogenics refrigerators, optimization generally refers to maximizing the available refrigeration by systematically selecting regenerator parameters such as geometry, type of matrix design, and matrix material that achieve this goal. The critical parameters affecting the thermal performance of regenerator are the number of heat transfer units, the fluid heat capacity ratio, the matrix heat capacity ratio, the thermal losses such as the longitudinal conduction. [4]

These parameters establish the thermal performance of a regenerator because they determine the temperature difference between the fluid and the matrix, the temperature swing of the matrix material, and any other irreversible heat transfer processes that contribute to degradation in regenerator performance. To maximize thermal performance both the NTU and matrix capacity ratio must be made as large as possible. However, in designing a regenerator for an actual cryogenic refrigerator the major obstacle limiting the magnitude of these parameters is the additional requirement to keep the pressure drop and regenerator void volume small. It is these conflicting requirements that lead to the need for regenerator optimization in a cryogenic refrigerator and a thorough understanding of the interaction of all key parameters. Walker (1973) describes the optimization problem for the designer as the task of satisfying the following conflicting requirements:

1) The temperature swing of the matrix must be minimized. Thus, the matrix heat capacity ratio must be a maximum. This can be achieved by a large, solid matrix.
2) The pressure drop across the regenerator must be small. The effect of the pressure drop across the across the matrix is to reduce the magnitude of the pressure excursion in the expansion space, thereby reducing the area of the expansion-space PV diagram and the gross refrigeration produced by the refrigerator. The pressure drop is minimized by a small (short), highly porous matrix.
3) A third consideration is the void volume. For a fixed-volume refrigerator such as the Stirling cycle refrigerator, the void volume influences the ratio of the maximum-to-minimum volume of the working space, which directly affects the pressure excursion in the expansion space. For maximum refrigeration, the pressure ratio must be large, or the void volume small. This can be achieved by a small, dense matrix. [4]

# OPTIMIZATION ANALYSIS

To illustrate the conflicting requirements that occur in the optimization of a regenerator for a cryogenic refrigerator, we shall consider a procedure for maximizing the available refrigeration in Stirling cycle refrigerator by optimizing typical regenerator design. The Stirling-Cycle refrigerator operates between an ambient temperature of 300 K and a refrigeration temperature of 80 K. The objective of the optimization is to maximize the available refrigeration by determining the values of the key regenerator parameters – such as the length, matrix material, and porosity – required to maximize the performance of the regenerator. The optimization is performed given the following operating condition:

1) The temperature difference across the regenerator is 300 K to 80 K.
2) The frequency of operation of the cryocooler is fixed.
3) The mean operating pressure of the refrigerator is fixed.
4) The piston and displacer motions are sinusoidal. [4]

The equation expressing the maximum available refrigeration, Wnet is obtained through a decoupled energy summation of the gross refrigeration produced by the cryocooler, less the individual losses limiting the net available refrigeration. The decoupled approach has been shown to provide accurate results by Harris, Rios, and Smith (1971) in their paper on regenerators for Stirling-type refrigerators. The energy summation is:

$$W_{net} = W_{pv} – (W_{\Delta p} + Q_{reg} + Q_c + \sum Q_1)$$

Where the individual losses described above consist of those associated with the regenerator – such as the pressure drop loss $W_{\Delta p}$, the thermal loss of the regenerator Qreg and the solid conduction loss through the matrix material $Q_c$ – and those associated with other cryocooler components, $\sum Q_1$. The regenerator losses expressed above are defined by the thermodynamics and dynamic equations. [4]

GROSS REFRIGEARTION [4]

The gross refrigeration is defined as the cyclic integral of the maximum cycle pressure and expansion volume variations:

$$W_{pv} = \int P^* dV_e$$

The equations for the maximum pressure and volume variations for Stirling cycle refrigeration will be presented latter in the report. [4]

PRESSURE DROP LOSS [4]

The pressure drop loss is the loss in refrigeration resulting from the pressure difference between the compression and expansion spaces. The loss in pressure in the expansion space is produced by both the frictional pressure drop through the regenerator and the pressure drop caused by the filling of the regenerator void volume during the pressurization and de-pressurization of the regenerator,

$$\Delta p = \Delta p_f + \Delta p_{pv}$$

And the loss in refrigeration is:

$$W_{\Delta p} = \int p_c * dV_e - \int (p_c - \Delta p) * dV_e$$

Or,

$$W_{\Delta p} = +\int \Delta p * dV_e$$

REGENERATOR THERMAL LOSS

The regenerator thermal loss is expressed in terms of the regenerator effectiveness, fluid thermal capacity, and the temperature difference across the regenerator.

$$Q_{reg} = + (1\text{-}Er) * m * cp * (T_w - T_c) * \lambda_h$$

LONGITUDNAL CONDUCTION LOSS

The longitudinal conduction loss is defined in terms of the matrix cross-section area-to-length ratio, thermal conductivity, and the temperature difference across the regenerator. In addition, the conduction loss occurs over the total period of operation and therefore is expresses as:

$$Q_c = +[A_m/L * \int (k * dT)_m] * (\lambda_h + \lambda_c)$$

Or,

$$Q_c = +2 * [A_m/L * \int (k * dT)_m] * \lambda_h$$

(For a balanced heat flow, $\lambda_h = \lambda_c$)

Experimental results for wire screen matrices have shown that longitudinal conduction for many commonly used matrix materials is controlled primarily by the interfacial resistance between screens. It can be represented by the following equation:

$$Km = 0.7 * (Tm/300\ K)$$

Thus thermal conduction can be expressed in terms of average thermal conductance over the temperature range from 80 to 300 K as:

$$Q_c = +2 * \lambda_h * K_m * (A_m/L) * (T_w - T_c)$$

**OPTIMIZATION EQAUTIONS:**

Expressing the optimization equation as the sum of the ratio of the individual energy loss terms to the gross refrigeration, we obtain the dimensionless equation that defines the terms to be minimized in order maximize the available refrigeration:

$$(W_{net} + Q_{reg} + W_{\Delta p} + Q_c + \sum Q_1)/W_{pv} = 1$$

The relations between the key parameters in the above equation are defined below using the classical Schmidt analysis for a Stirling Cycle refrigerator. The approach of the Schmidt analysis is to specify simple sinusoidal motion for the compressor piston and the displacer and to assume isothermal compression and expansion processes. As the expansion process is assumed to be isothermal, the maximum refrigeration produced is equal to the gross expansion work performed:

$$Q_r = m*q_r*\lambda_c = W_{pv}$$

Where qr is the heat transferred to the working fluid per cycle in the expansion space and Ps is the system pressure, considered here as the maximum working pressure in the refrigerator. For an ideal Stirling cycle refrigerator with isothermal compression and expansion processes, the heat transferred to the working fluid is given by:

$$q_r = R*T_c*\ln(Pa)$$

Also, as the motion is considered sinusoidal, a phasor diagram can be used to describe the variations of the compression and expansion volumes and to assist in the development of the equations describing these variations. The advantage of using the phasor notation is that the amplitude and phase relationship of each of the system operating parameters can be presented on a phasor diagram that provides physical insight into the mathematics of the Schmidt analysis (Ackermann, 1981).

GROSS REFRIGERATION EQUATION [4]

The system pressure Ps and the expansion volume derived from Schmidt analysis of phasor equations are as follows:

$$P_s = P_{mean} + p_{so}*\sin(\omega t + \beta)$$

$$V_e = ½(V_e)_{max} + v_{eo}*\sin(\omega t)$$

From the above pressure equations, gross refrigeration can be determined by substituting the differentiated expansion volume and the maximum system pressure equation in the gross refrigeration equation.

$$W_{pv} = \int [\, P_{mean} + p_{so}*\sin(\omega t + \beta)]*(\omega*v_{eo}*\cos(wt))dt$$

And integrating over the complete cycle gives:

$W_{pv} = \pi * p_{so} * v_{eo} * \sin(\beta)$

PRESSURE DROP EQUATIONS [4]

The final compression and expansion space pressures are found by including the pressure drop. The pressure drop has the effect of reducing both the magnitude of the pressure in the expansion space and the phase angle by which the expansion space pressure leads the expansion space volume variations. Both effect result in a reduction in the refrigeration produced by the cryocooler. The total pressure drop is the sum floe frictional pressure drop across the regenerator, Δpf, and the reduction in the system pressure phasor caused by the pressurization and de-pressurization of the regenerator void volume, Δprv. From the analysis of the phasor diagram we obtain:

$$\Delta p_f = -(\Delta p_f)_0 * \sin[\omega t + (\beta + \Upsilon)]$$

Where ϒ is the angle between the pressure drop phasor and the system pressure phasor.

And,

$$\Delta p_{rv} = -(\Delta p_{rv})_0 * \sin(\omega + \beta)$$

Substituting these two pressure drop components, and the differentiated expansion volume into the pressure drop loss equation and integrating it over a cycle, we obtain:

$$W_{\Delta p} = + \pi * v_{eo}[(\Delta p_f)_0 * \sin(\beta + \Upsilon) + (\Delta prv)_0 * \sin(\beta)]$$

And the ratio of pressure drop loss to the gross refrigeration is defined in terms of the pressure drop amplitude $(\Delta pf)_o$ and $(\Delta prv)_o$ as:

$$W_{\Delta p}/W_{pv} = + [(\Delta_{pf})_o * \sin(\beta + \Upsilon)]/p_{so} * \sin(\beta) + (\Delta p_{so})o/p_{so}$$

REGENERAOR FRICTION PRESSURE DROP [4]

The amplitude of the frictional pressure drop is determined from the Fanning pressure drop equation:

$$(\Delta p_f)o = f * (G^2/2\rho) * (L/r_h)$$

Where G is the average mass flow rate per unit of the free flow area of the fluid, out of the regenerator and into the expansion space during the heating flow period and out of the expansion space and in to the regenerator during the cooling flow period:

$$G = (me/A_{ff}) = 1/\lambda_c * \int(\rho_f)e * w * dt$$

This equation provides a means for evaluating pressure drop given the flow velocity and matrix geometry. However, it does not relate the pressure drop to the heat transfer characteristics of the regenerator as required to facilitate the optimization procedures. To achieve this , Kays and London (1964) present a useful correlation between the dimensionless Stanton number and the friction factor. A geometric factor can be defined in terms of Stanton number and friction factor, or equivalently in terms of the total regenerator NTU:

$$\Gamma = St*Pr/f = 2*(NTU)*Pr^{2/3}/f*(r_h/L)$$

where the relationship between the NTU and St is :

$$NTU = St/2*(L/r_h)$$

Substitution of this expression into the pressure drop equation produces an optimization equation for the frictional pressure drop in terms of NTU parameter and the geometric factor:

$$W_{\Delta pf}/W_{pv} = [(NTU*pr^{2/3}/\Gamma)*(G^2/\rho)*\sin(\beta+\Upsilon)]/p_{so}*\sin(\beta)$$

From the above equation we can see that to minimize the pressure drop loss, we must minimize the NTU/Γ ratio or equivalently, for a given geometry, minimize the NTU, which is opposite to what must be done to reduce the regenerator thermal loss.

<u>REGENERATOR VOID VOLUME PRESSURE DROP [4]</u>

The pressure drop related to the regenerator void volume is the difference between the amplitude of the system pressure phasor with no void volume and the amplitude of the pressure with regenerator void volume:

$$(\Delta p_{rv}) = p_{so} - p*_{so}$$

where the star denotes the pressure amplitude with regenerator void volume. To determine the void volume pressure ratio in the work loss due to pressure drop equation, the system pressure amplitude is derived in terms of the mean cycle pressure and the pressure ratio between the maximum and minimum cycle pressures,

$$p_{so} = P_{mean} - P_{min} = P_{max} - P_{mean}$$

which can also be expressed in terms of the pressure ratio:

$$p_{so} = P_{mean}*(Pa-1)/Pa+1)$$

where Pa is the pressure ratio defined as $P_{max}/P_{min}$. Substitution of this expression or the system pressure amplitude in the first equation produces the following expression for the pressure ratio term in the optimization equation:

$$(\Delta p_{rv})o/p_{so} = [(Pa-1)/Pa+1) - (Pa*-1)/(Pa*+1)]/[(Pa-1)/Pa+1)]$$

where Pa* denotes the pressure ratio with regenerator void volume.

If we now use Schmidt analysis to define the pressure ratio in terms of the volume and temperature system parameters, we see that the void volume pressure ratio is a function of the void volume of the regenerator, and is expressed in terms of void volume to expansion volume space. Thus ,for a given temperature and compression-to-expansion-volume ratio, the pressure loss from the regenerator void volume is a function of only void volume ratio, Vrv/Ve, and increase asymptotically to 1 as the regenerator void volume ratio increases:

$$W_{\Delta prv}/W_{pv} = (\Delta p_{rv})o/p_{so} = (V_{rv}/V_e)/[1/2*(1+T_c/T_w)*(T_w/T_c+V_{cs}/V_e)+(V_{rv}/V_e)]$$

where the subscript are as follows: w is the warm temperature, c is the cold temperature, cs is the compression space, e is the expansion space, and rv is the regenerator void volume.

# OPTIMIZATION PROCEDURE

From the above equations we see that to optimize the regenerator in the Stirling cycle cryocooler we must determine the regenerator parameters – length, frontal area, matrix geometry, matrix material, and porosity – that produce the optimum NTU and matrix capacity ratio, and minimize the longitudinal thermal conduction and pressure drop. Also, the optimization of these parameters must be consistent with constraints imposed on the cryocooler design, which for our example are:

1) The temperature difference across the regenerator is 300 K to 80 K.
2) The operating frequency of the cryocooler is 50 Hz.
3) The mean operating pressure of the refrigerator is 1.5 bar.
4) The piston and displacer motions are sinusoidal.
5) The nominal displacer stroke is

With these constraints, and the idealization of the Stirling cycle cryocooler, the optimization equations are:

1) THE REGENERATOR THERMAL LOSS

$$Q_{reg}/W_{pv} = (1\text{-}Er)*(T_w\text{-}T_c)*(cp)f/q_r = Ie*(T_w\text{-}T_c)(cp)f/q_r$$

Where, considering an ideal Stirling cryocooler, the heat transferred to the working fluid is given by:

$$q_r = R*Tc*\ln(Pa)$$

and the regenerator loss ratio is:

$$Q_{reg}/W_{pv} = Ie*[(T_w\text{-}T_c)/T_c]*[(cp)_f/R(\ln(Pa))]$$

Also, as the refrigeration in a regenerative cycle cryocooler only occurs during one-half of the cycle, the average mass flow rate through the regenerator during each period is:

$$m = W_{pv}/q_r*\lambda_c = 2*W_{pv}/q_r = [2*\pi*fr*p_{so}*v_{eo}*\sin(\beta)]/q_r$$

$$= [\pi*fr*p_{so}*Ve*\sin(\beta)]/R*T_c*\ln(Pa)$$

where fr is the frequency of operation of the cryocooler.

2) THE LONGITUDINAL CONDUCTION LOSS

$$Q_c/W_{pv} = [\lambda*Km*Am*(T_w\text{-}T_c)/Lf]/m*R*Te*\ln(Pa)$$

3) THE FRICTIONAL PRESSURE DROP LOSS\

$$W_{\Delta pf}/W_{pv} = [(NTU*Pr^{2/3}/\Gamma)*(G^2/\rho)*\sin(\beta+\Upsilon)]/p_{so}*\sin(\beta)$$

4) THE VOID VOLUME PRESSURE DROP LOSS

$$W_{\Delta prv}/W_{pv} = (V_{rv}/V_e)/[1/2*(1+T_c/T_w)*(T_w/T_c+V_{cs}/V_e)+(V_{rv}/V_e)]$$

The two additional equations required to obtain a solution are:

5) THE NUMBER OF HEAT TRANSFER UNITS

$$NTU = St*(L/r_h)*1/2$$

Where the Stanton number is derived from Kays and London's experimental data (1964) as $St*Pr^{2/3} = 0.68*Re^{-0.4}$

6) THE MATRIX HEAT CAPACITY RATIO

$$Cr/C_{min} = (M*cp)_m/(m*c_p)_f*\lambda_h$$

To illustrate the computational processes involved in optimizing a regenerator we connsider the optimization of a regenerator for a Stirling cryocooler. The cryocooler is required to produce 0.250 Watts of cooling at 80 K and will use a wire screen regenerator matrix. The design conditions and thermal properties for this cryocooler are summarized in the 1st table on the next page.

The three assumptions in the 1st table regarding the regenerator geometry draw on previous experience with Stirling cycocoolers and wire screen regenerators. As we will see as we proceed with the optimization, the requirement for good, first-order assumptions is critical to the process in order to minimize the number of iterations required to arrive at a solution. The optimization process is not a numerically automated process that will converge to the solution based on the constraints and input parameters; instead it will be an interactive process between the designer and the available regenerator performance data.

To perform the optimization, we begin by assuming some reasonable loss values for the individual loss ratios, and from these values we determine the NTU, matrix capacity ratio, and from these values we determine the NTU, matrix capacity ratio, and void volume ratio. If the values obtained are unrealistic, or if the design can be further optimized to reduce the loss terms, then additional iterations are required to further optimize the regenerator. If the values are reasonable, the designer can proceed with the cyocooler overall design to determine whether the regenerator design meets the other crycocooler requirements. A first estimation of losses is presented in the 2nd table.

From the values of the 1st and the 2nd table we can proceed to calculate the performance parameters and losses by first calculating the void volume ratio, and the expansion swept volume. With these valued in hand, all of the other regenerator dimensions can be determined and, in turn, the performance parameters calculated.

TABLE 1: STIRLING CYCLE REFRIGERATOR DESIGN PARAMETERS [4]

| PARAMETER | VALUE |
|---|---|
| **Input parameters** | |
| | |
| Cycle | Stirling |
| Working Fluid | Helium |
| Warm Temperature ($T_w$) | 300 K |
| Cold Temperature ($T_c$) | 80 K |
| Net Refrigeration ($W_{net}$) | 250 mW |
| Frequency (fr) | 50 Hz |
| Regenerator Material | Bronze screen |
| Screen Mesh | 400*400 |
| Wire Diameter (dw) | 0.05 mm |
| Geometric Factor | 0.08 |
| Mean Pressure ($P_{mean}$) | 25 bar |
| | |
| **Properties** | |
| | |
| Prandtl Number (Pr) | 0.75 |
| Regenerator Porosity ($\alpha$) | 0.65-0.72 |
| Matrix Density ($\rho_m$) | 8.7 $gm/cm^3$ |
| Matrix specific heat | 0.30 J/g-K |
| Mean matrix thermal conductance ($K_m$) | 7.62 MW/cm-K |
| Mean regenerator helium density ($\rho f$) | 0.005 $g/cm^3$ |
| Mean expansion space helium density ($\rho_{ef}$) | 0.009 $g/cm^3$ |
| Helium specific heat (cpf) | 5.2 J/g-K |
| Mean helium thermal conductivity ($K_f$) | 1.0 MW/cm-K |
| Mean Viscosity ($\mu f$) | $1.44*10^{-4}$ g/cm-s |
| Gas constant (R) | 2.08 J/g-K |
| | |
| **Assumptions** | |
| Swept volume ratio ($V_{cs}/V_e$) | 5.0 |
| Compression-to-expansion-volume phase angle ($\phi$) | 90 |
| Regenerator matrix porosity ($\alpha$) | 0.67 |

TABLE 2: FIRST ESTIMATE OF LOSSES IN A STIRLING CRYCOOLER [4]

| **Loss parameter** | **Value** |
|---|---|
| | |
| Regenerator thermal loss ($Q_{reg}/W_{pv}$) | 0.20 |
| Frictional pressure drop loss ($W_{\Delta pf}/W_{pv}$) | 0.10 |
| Void volume pressure loss ($W_{\Delta prv}/W_{pv}$) | 0.51 |
| Thermal conduction loss ($Qc/W_{pv}$) | 0.05 |
| Additional thermal and mechanical losses ($\sum Q/W_{pv}$) | 0.12 |
| Total losses | 0.98 |
| Net refrigeration ($W_{net}/W_{pv}$) | 0.02 |

TABLE 3: CALCULATED REGENERATOR PARAMETERS FROM THE OPTIMIZATION EQUATIONS [4]

| **Parameter** | **Value** | **Formula** |
|---|---|---|
| | | |
| Void volume ratio ($V_{rv}/V_e$) | 5.76 | $W_{\Delta prv}/W_{pv}$ |
| Pressure ratio | 0.35 | |
| Pressure amplitude ($p_{so}$) | 2.07 bar | $p_{so} = P_{mean}*(Pa-1)/(Pa+1)$ |
| Average mass flow rate (m) | 0.00020556 kg/sec | $m = 2*W_{pv}/R*T_c*\ln(Pa)$ |
| Expansion swept volume ($V_e$) | 2.2476E-07 $m^3$ ($\beta$ = 126) | $W_{pv} = \pi/2*fr*p_{so}*V_e*\sin(\beta)$ |
| Void Volume ($V_{rv}$) | 1.2964E-06 $m^3$ | $V_{rv} = 1.85*V_e$ |
| Regenerator volume ($V_r$) | 1.9394E-06 $m^3$ | $V_r = V_{rv}/\alpha$ |
| Matrix volume ($V_m$) | 6.3852E-07 $m^3$ | $V_m = (1-\alpha)*V_r$ |
| Expander stroke (S) | 1.00 cm | |
| Displacer diameter ($D_d$) | 0.0053509 m | |
| Regenerator diameter ($D_r$) | 0.0048509 m | $D_r = D_d – 2*t$ |
| Regenerator length (L) | 0.075 m | |
| Regenerator matrix mass ($M_m$) | 0.00555515 kg | $M_m = (\rho V)_m$ |
| Regenerator free flow area ($A_{ff}$) | 1.2376E-05 $m^2$ | $A_{ff} = \alpha*A_r$ |
| Mass flow/area (G) | 16.6091374 kg/$m^2$ | $G = m/A_{ff}$ |
| Regenerator area ($A_r$) | 1.8472E-05 $m^2$ | |
| Number of heat transfer units (NTU) | 153.69735 | $W_{\Delta pf}/W_{pv}$ |

TABLE 4: CALCULATED REGENERATOR HEAT TRANSFER PARAMETERS FROM OPTIMIZED REGENERATOR VLAUES [4]

| **Parameter** | **Value** | **Formula** |
|---|---|---|
| | | |
| Hydraulic radius ($R_h$) | 2.5379E-05 m | $D_h = \alpha * d_w/(1-\alpha)$ |
| Reynolds number (Re) | 117.088826 | $Re = G * D_h/\mu$ |
| Stanton number (St) | 0.12257311 | St*Pr2/3 = 0.68*Re-0.4 |
| Heat transfer area (A) | 0.03657459 $m^2$ | $A = (L/r_h) * A_{ff}$ |
| Nusselt number (Nu) | 10.7742836 | Nu = 0.68*Re0.6*Pr0.33 |
| Number of heat transfer units (NTU) | 181.115498 | $NTU = St(L/R_h) * 0.5$ |
| Matrix capacity ratio ($Cr/C_{min}$) | 155.911425 | $C_r/C_{min} = (Mcp)m/(mcp)_f * \lambda_h$ |

TABLE 5: CALCULATED REGENERATOR LOSSES [4]

| **Loss component** | **Loss value (W)** |
|---|---|
| | |
| Regenerator thermal loss ($Q_{reg}$) | 1.17579653 |
| Thermal conduction loss ($Q_c$) | 0.0190297 |
| Friction pressure drop loss ($W_{pf}$) | 1.47298813 |
| Void volume pressure loss ($W_{prv}$) | 6.375 |
| Additional thermal losses ($\sum Q$) | 3.125 |
| Net refrigeration ($W_{net}$) | 0.33218563 |

# GRAPHS

A) Regenerator Porosity V/S ($Q_c$, $Q_{reg}$, $W_{pf}$, $W_{prv}$)

Fig. 20: REGENERATOR POROSITY V/S $Q_c$ (Watts)

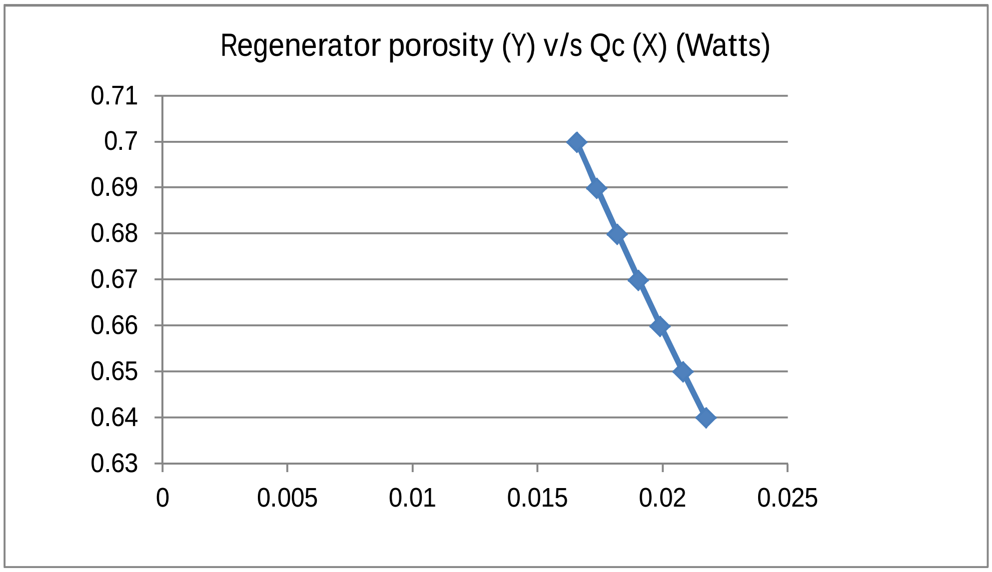



Fig. 21: REGENERATOR POROSITY (Y) V/S $Q_{reg}$ (X) (Watts)

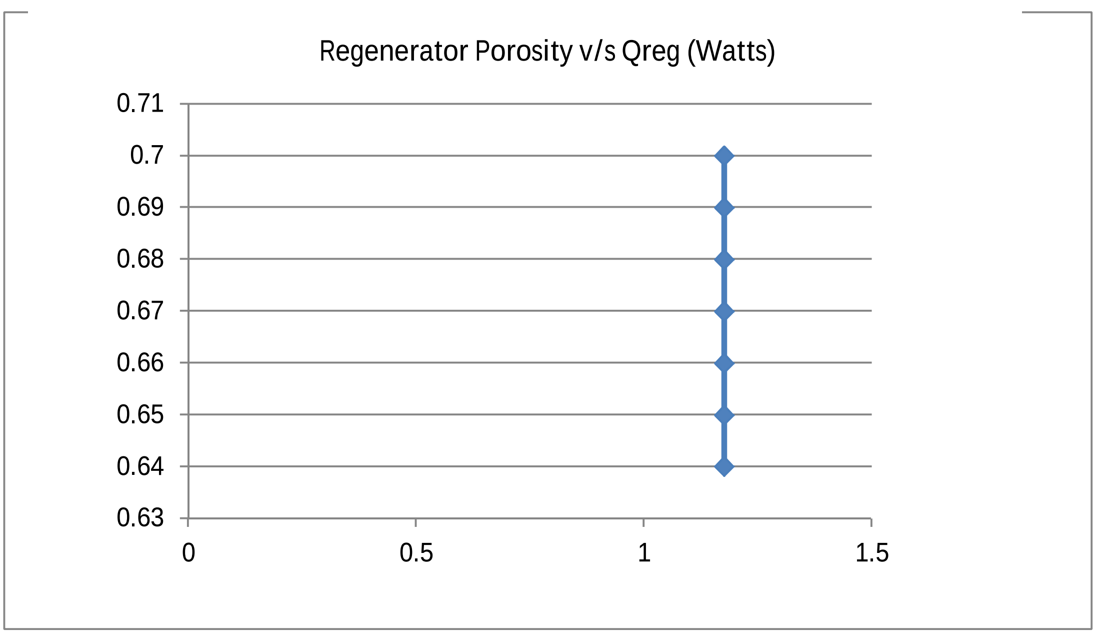



Fig. 22: REGENERATOR POROSITY (Y) V/S $W_{pf}$ (X)(Watts)

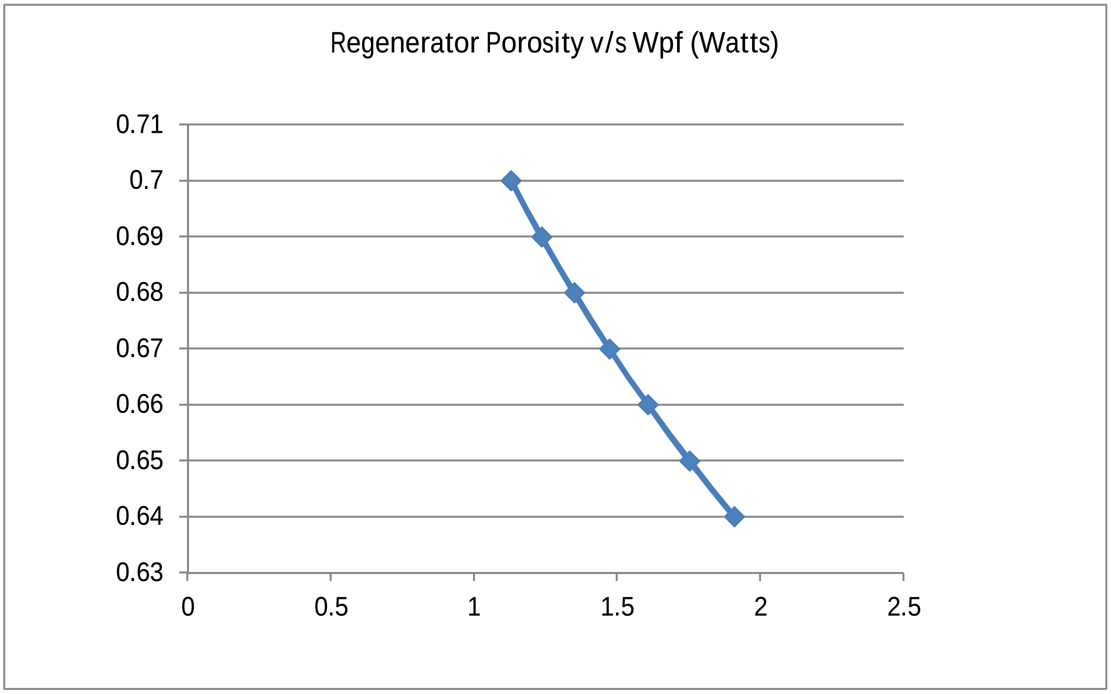
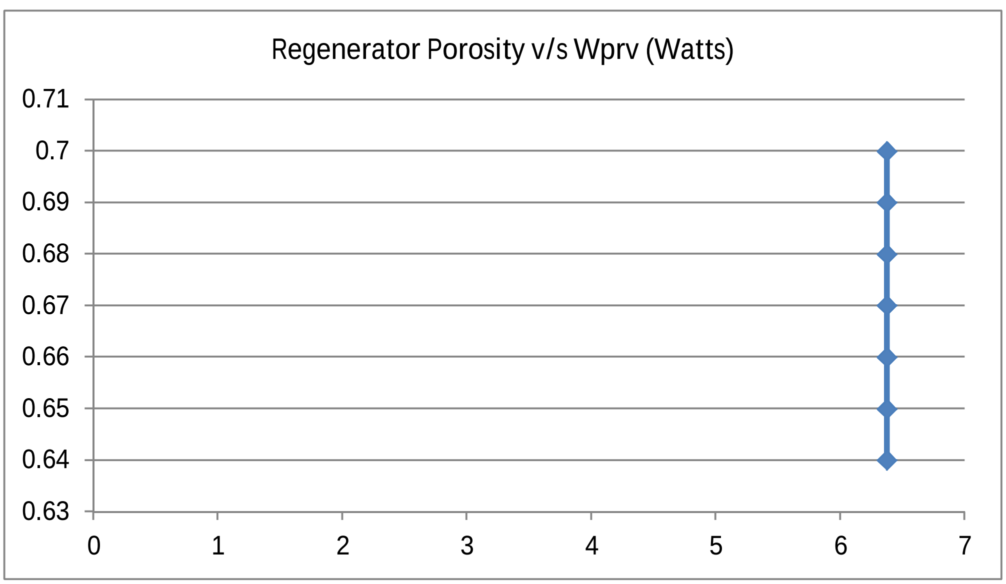



Fig. 23: REGENERATOR POROSITY (Y) V/S $W_{prv}$ (X)(Watts)




B) Frequency V/S ($Q_c$, $Q_{reg}$, $W_{pf}$, $W_{prv}$)

Fig. 24: FREQUENCY (Y) V/S $Q_c$ (X)(Watts)

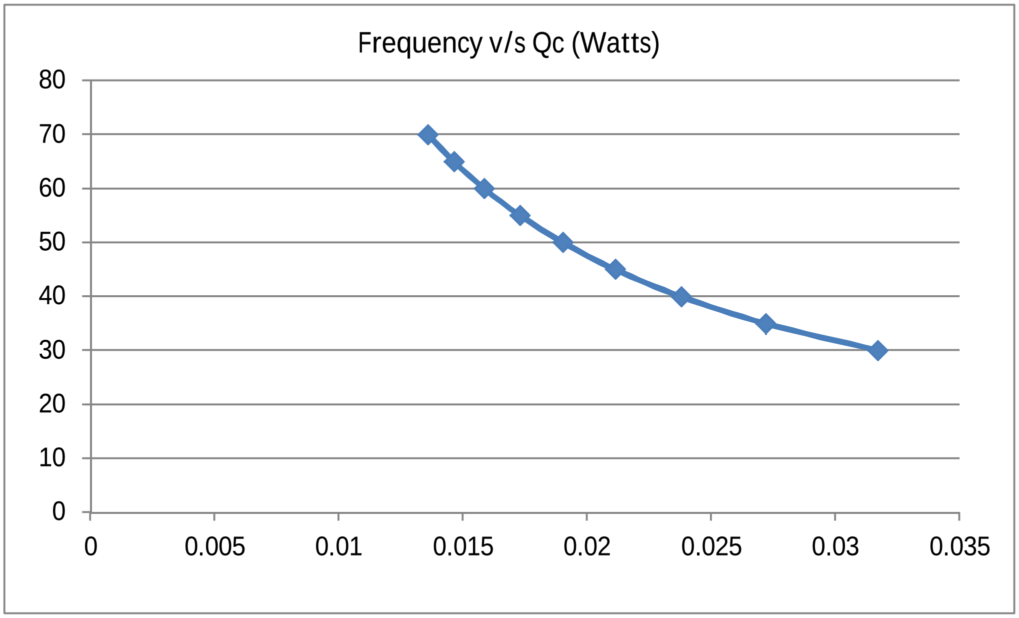



Fig. 25: FREQUENCY (Y) V/S $Q_{reg}$ (X)(Watts)

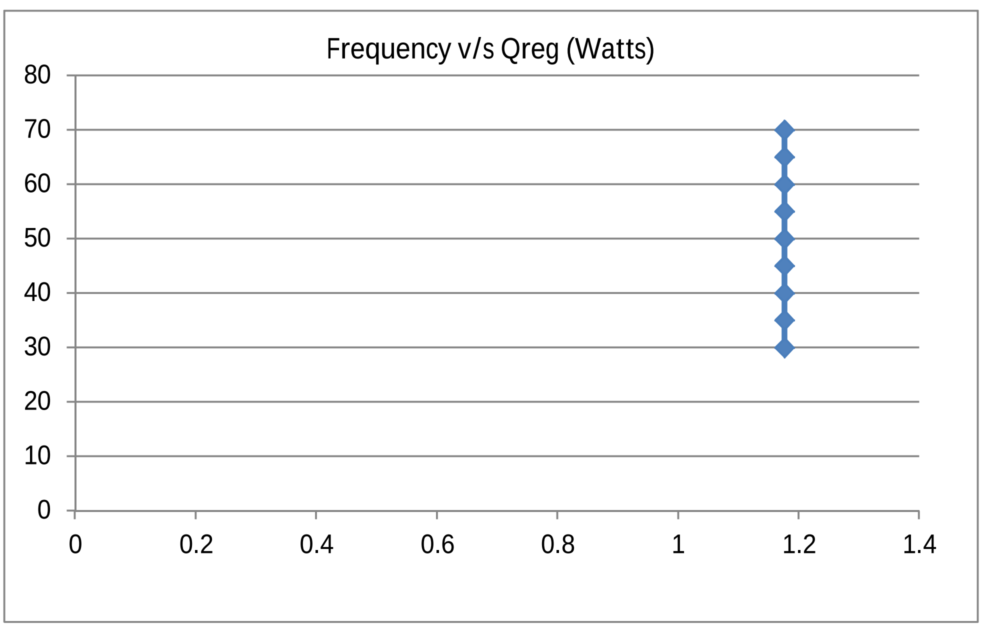



Fig. 26: FREQUENCY (Y) V/S $W_{pf}$ (X)(Watts)

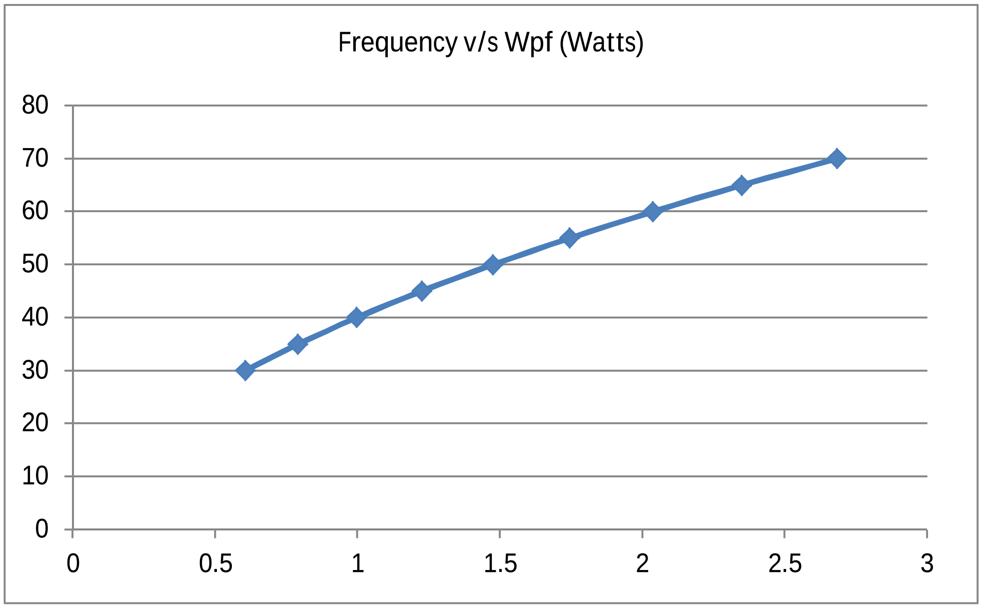
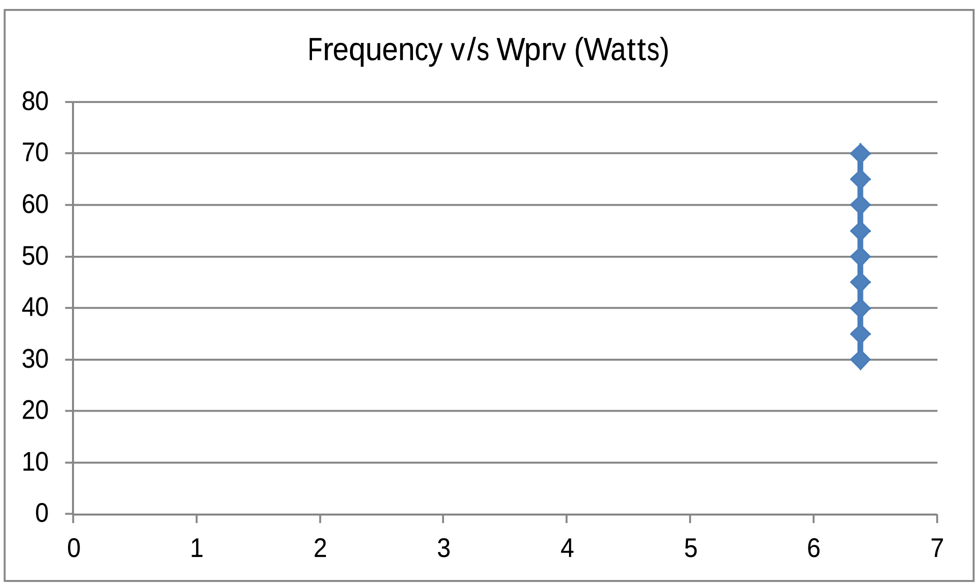



Fig. 27: FREQUENCY (Y) V/S $W_{prv}$ (X)(Watts)



C) Tc v/s ($Q_c$, $Q_{reg}$, $W_{pf}$, $W_{prv}$)

Fig. 28: $T_c$ (Y)v/s $Q_c$ (X)(Watts)

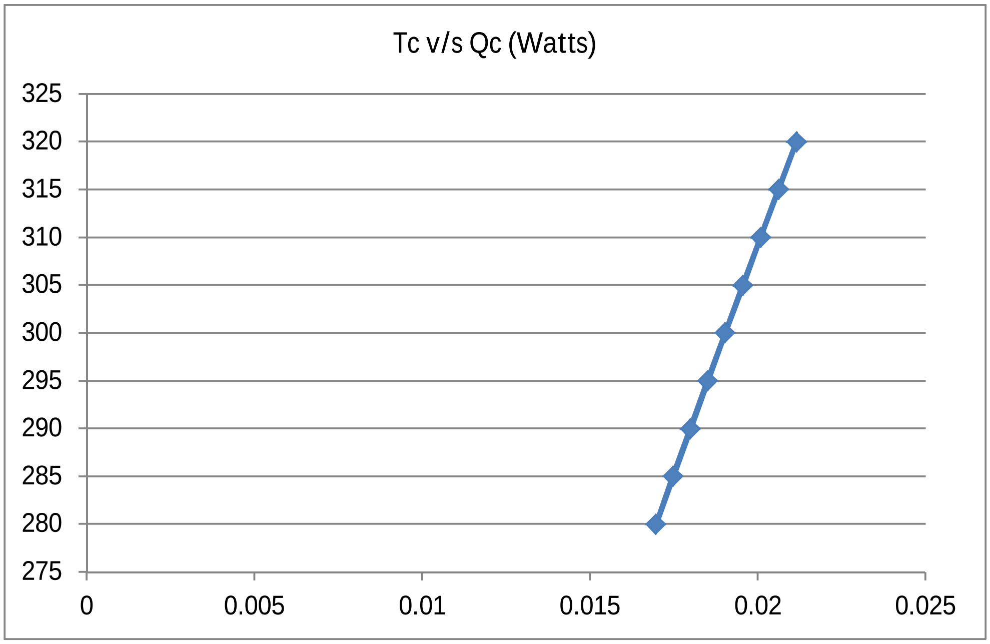



Fig. 29: $T_c$ (Y)v/s $Q_{reg}$ (X) (Watts)

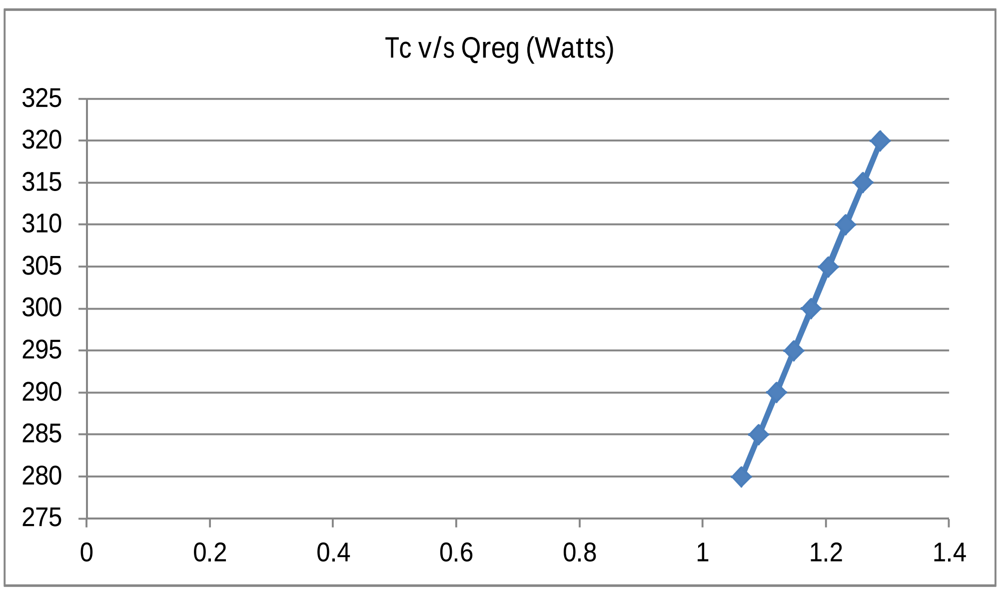


Fig. 30: $T_c$ (Y)v/s $W_{pf}$ (X)(Watts)

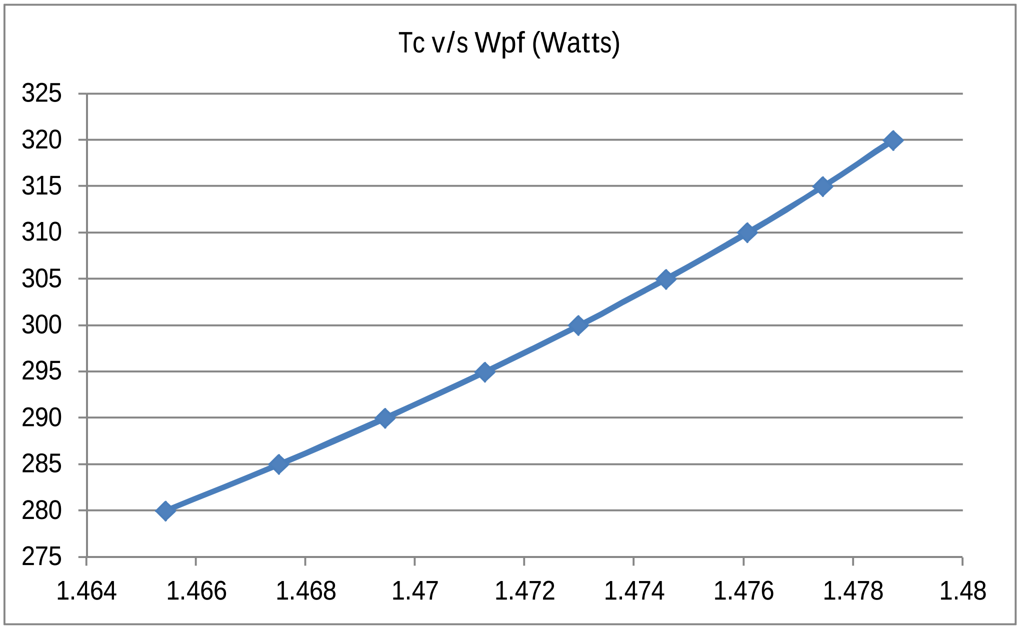
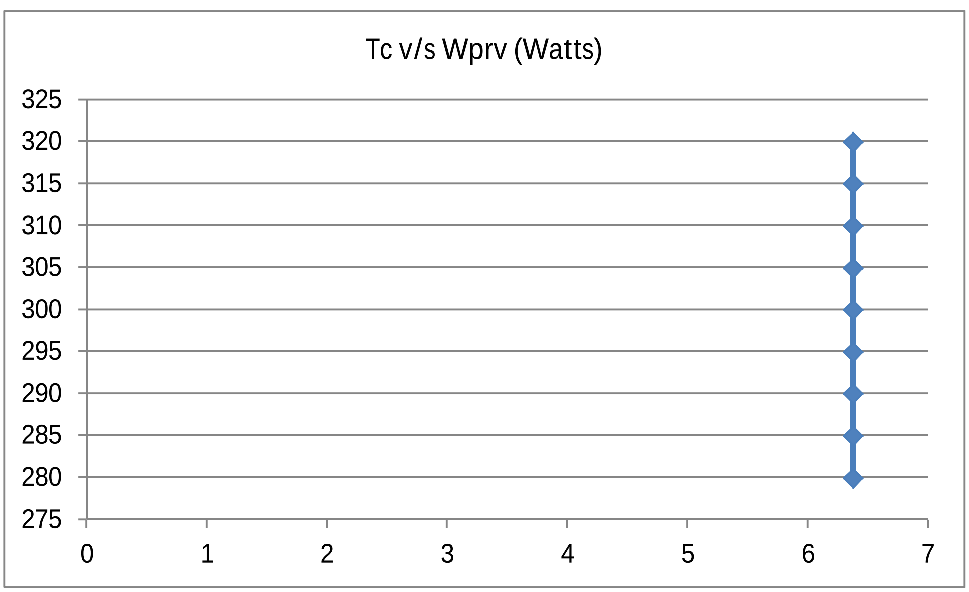



Fig. 31: $T_c$ (Y)v/s $W_{prv}$ (X)(Watts)



D) VOLUME RATIO ($V_c/V_e$) V/S ($Q_c$, $Q_{reg}$, $W_{pf}$, $W_{prv}$)

Fig. 32: VOLUME RATIO ($V_c/V_e$) (Y) v/s $Q_c$ (X)(Watts)

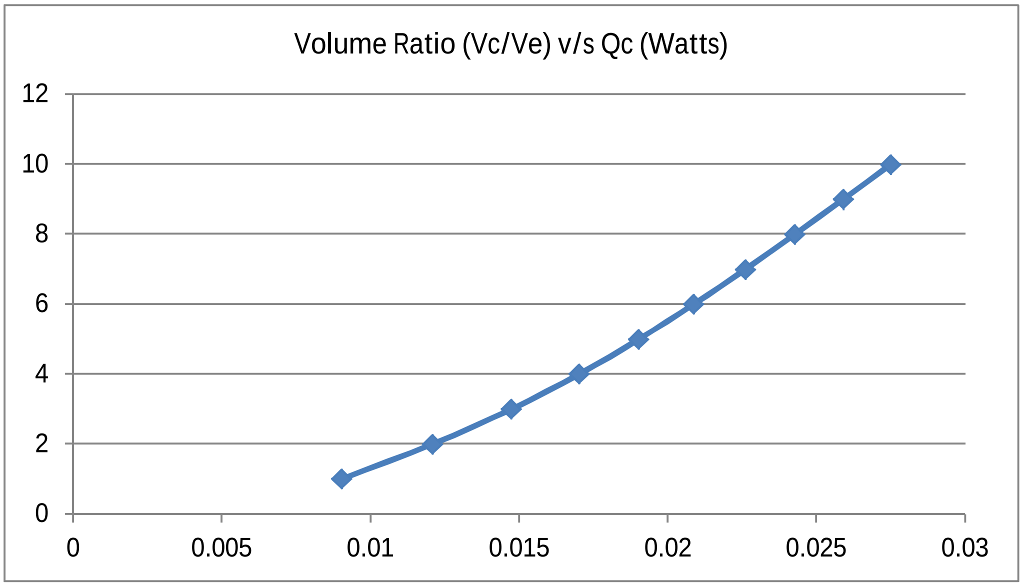



Fig. 33: VOLUME RATIO ($V_c/V_e$) (Y) v/s $Q_{reg}$ (X)(Watts)

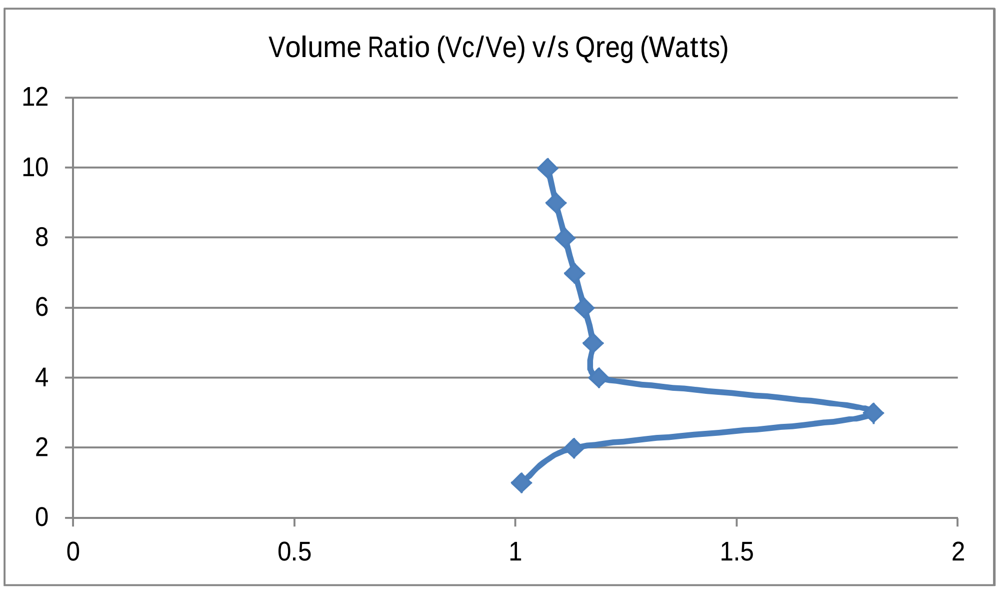


Fig. 34: VOLUME RATIO ($V_c/V_e$) (Y) v/s $W_{pf}$ (X)(Watts)

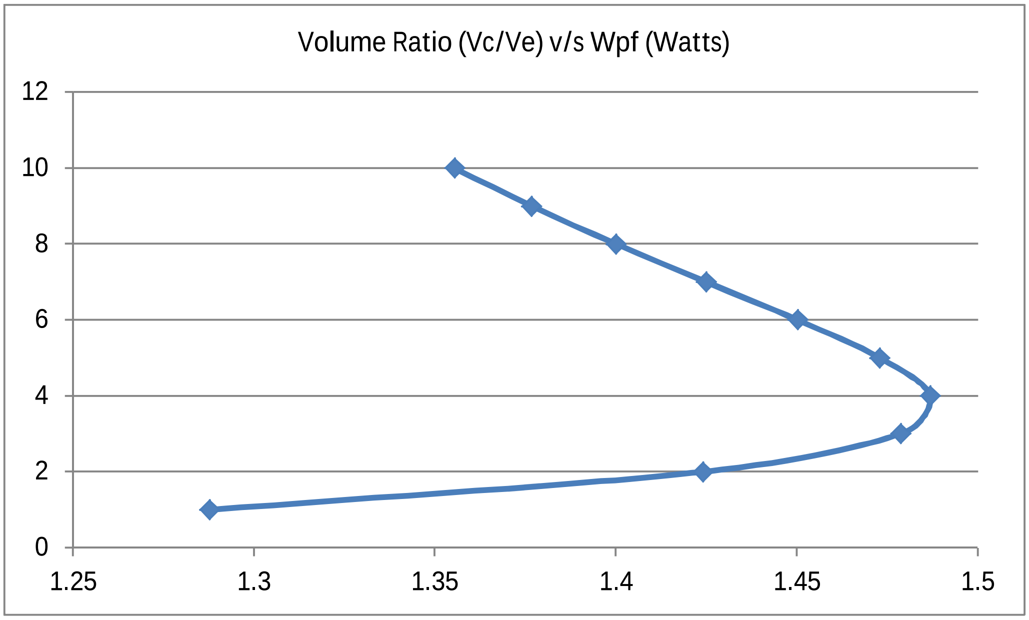
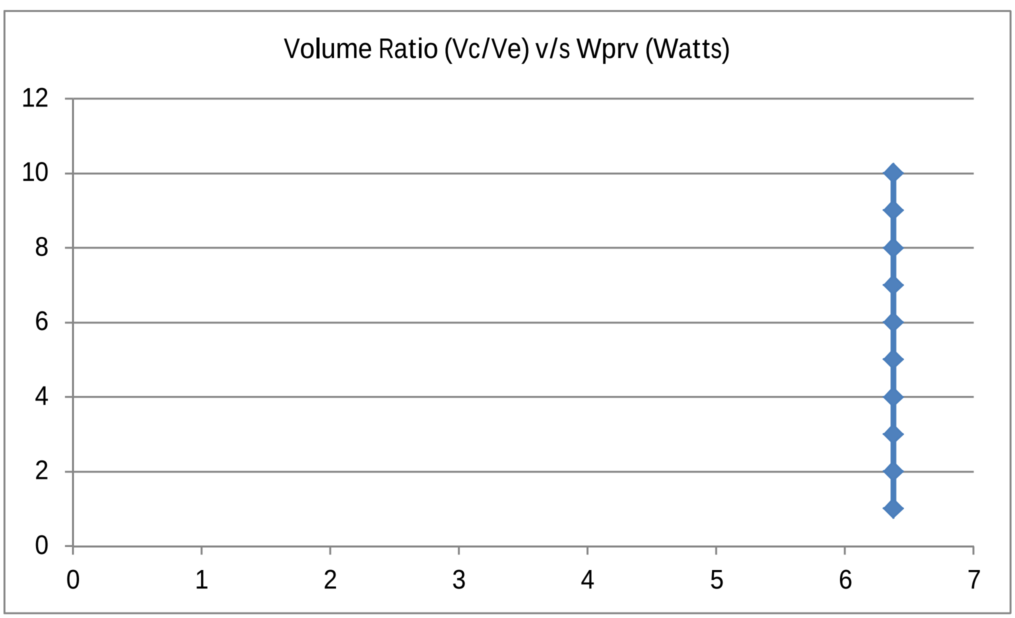



Fig. 35: VOLUME RATIO ($V_c/V_e$) (Y) v/s $W_{prv}$ (X)(Watts)



# CONCLUSION

In this report we have reproduced the analysis given by Robert Ackermann for a miniature Stirling cryocooler. We changed a few input parameters according to our needs. By assuming the relative ratio of different losses, we obtained regenerator parameters from optimization equations. With the help of these parameters we calculated the actual loss's taking place inside the cyocooler. The actual refrigeration obtained from this analysis comes out to be 332.18 mW which is close to the theoretically assumed value of 250 mW.

The design parameters were obtained from optimization equations. By obtaining the expander stroke length (S) to 1.00 cm and assuming regenerator length to be 7.50 cm, the displacer diameter ($D_d$) and regenerator diameter ($D_r$) are found to be:

1) $D_d$ = 5.35 mm
2) $D_r$ = 4.85 mm

Now with the help of expander stroke (S) and displacer diameter (Dd), we can calculate expansion volume (Ve). From this compression volume (Vc) can be calculated with help of ratio Vc/Ve which is already known.

The value of $C_L/C_D$ for a compressor is assumed to be 1.

Where,

$C_L$ = Length of compressor stroke

$C_D$ = Diameter of compressor

Thus with the help of above relation, the compressor length and compressor diameter are obtained as:

$C_L$ = 1.12 cm

$C_D$ = 1.12 cm

# REFERENCES


1) Open source reference: www.Wikipedia.com

2) Robert Sier (1999). *Hot air caloric and stirling engines. Vol.1, A history* (1st Edition (Revised) edition)

3) Graham Walker, Stirling Cryocoolers, Cryocoolers Part-1, page 95-142

4) Robert Ackermann, Cryogenic Regenerative Heat Exchangers, Page 147-191

5) Berchowitz, D.M., Rallis, C.J., and Urieli, I. (1977). "A New Mathematical Model for Stirling Cycle Machine." Proc. 12th I.E.C.E.C., pp. 1522-1527, Washington, D.C., August 28-September 2

6) Finkelstein, T., (1975). "Computer Analysis of Stirling Engines." *Adv. Cryog. Eng.* 20, 269-282

7) Finkelstein, T., (1978). "Modeling Design and Optimization of Stirling Engines." Proc. 13th I.E.C.E.C.., San Diego, California, August 20-25.

8) Gedeon, D. (1978). "Optimization of Stirling Cycle Machines."Proc. 13th I.E.C.E.C., San Diego, California

9) Kirk, A. (1874)."On the Mechanical Production of Cold." *Proc. Ins. Civil Eng*. 37, 244-315, London.

10) Kohler, J.W.L. (1965). The Stirling Refrigeration Cycle. *Sci. Am*. 212(4), 119-127

# APPENDIX

## PROGRAM NO. 1

### TITLE: Pressure V/S Total volume

```
TE = 80; TC = 300;
VE = 1*10^(-6);
Pmean = 25*10^(5);% 25 bar
alpha = 90;
x=1;
VC=VE*x;
y=2;
VD = y*VE;
t = TC/TE;
TD = (TE + TC)/2;
S = (VD*TC)/(VE*TD);
theta = atand((x*sind(alpha))/(t + x*cosd(alpha)));
A = sqrt (t^2 + 2*t*x*cosd(alpha) + x^2 );
B = t + x + 2*S;
delta = A/B;
Pmax = Pmean*sqrt((1+delta)/(1-delta));
disp(Pmax);
fi=0;
i=1;
while(fi<(361))
    p = Pmax*(1-delta)/(1+delta*cosd(fi-theta));
    Ve = 0.5*VE*(1+cosd(fi));
    Vc = 0.5*VC*(1+cosd(fi-alpha));
    V = VD + Vc + Ve;
    P(i)= p;
    Vt(i)= V;
    fi=fi+1;
    i=i+1;
end
disp(i);
fi=[0:1:360];
plot(Vt,P)
title('Pressure vs Total volume')
```

## PROGRAM NO. 2

### TITLE: Pressure vs Expansion and Compression volume

```
TE = 80; TC = 300;
VE = 1*10^(-6);
Pmean = 25*10^(5);% 25 bar
```

```
alpha = 90;
x=1;
VC=VE*x;
y=2;
VD = y*VE;
t = TC/TE;
TD = (TE + TC)/2;
S = (VD*TC)/(VE*TD);
theta = atand((x*sind(alpha))/(t + x*cosd(alpha)));
A = sqrt( t^2 + 2*t*x*cosd(alpha) + x^2 );
B = t + x + 2*S;
delta = A/B;
Pmax = Pmean*sqrt((1+delta)/(1-delta));
disp(Pmax);
fi=0;
i=1;
while(fi<(361))
    p = Pmax*(1-delta)/(1+delta*cosd(fi-theta));
    Ve = 0.5*VE*(1+cosd(fi));
    Vc = 0.5*VC*(1+cosd(fi-alpha));
    P(i)= p;
    Ve1(i)= Ve;
    Vc1(i)= Vc;
    fi=fi+1;
    i=i+1;
end
disp(i);
fi=[0:1:360];
subplot(2,1,1); plot(Ve1,P); title('Pressure vs Expansion volume');
subplot(2,1,2); plot(Vc1,P); title('Pressure vs Compression volume');
```

**PROGRAM NO.3**

**TITLE: ($P_{max}$, $P_{min}$, Qe, W, COP) v/s $T_C$**

```
TE = 80;
TC = 280;
VE = 1*10^(-6);    % 1 cc expansion volume
Pmean = 25*10^(5); % 25 bar
alpha = 90;
x=1;
VC=x*VE;
y=2;
VD = y*VE;
i=1;
while(TC<(330.1))
    t = TC/TE;
    TD = (TE + TC)/2;
    S = (VD*TC)/(VE*TD);
    theta = atand((x*sind(alpha))/(t + x*cosd(alpha)));
    A = sqrt( t^2 + 2*t*x*cosd(alpha) + x^2 );
    B = t + x + 2*S;
    delta = A/B;
    Pmax = Pmean*sqrt((1+delta)/(1-delta));
    Pmin = Pmean*sqrt((1-delta)/(1+delta));
    Q = (pi*Pmean*VE*delta*sind(theta))/(1 + sqrt( 1 - delta^2));
```

```
    Qc = (pi*Pmean*VE*x*delta*sind(theta-alpha))/(1 + sqrt(1-delta^2));
    w = Q - Qc;
    cop = Q/(Q - Qc);
    Pm1(i)= Pmax;
    Pm2(i)= Pmin;
    Qe(i)= Q;
    W(i)= w;
    COP(i)= cop;
    i=i+1;
    TC=TC+0.1;
end
disp(TC);
disp(i);
TC=[280:0.1:330];
subplot(5,1,4);
plot(TC,Pm1); title('Pmax vs Hot end temperature');
subplot(5,1,2);
plot(TC,Pm2); title('Pmin vs Hot end Temperature');
subplot(5,1,3);
plot(TC,Qe); title('Refrigeration vs Hot end Temperature');
subplot(5,2,1);
plot(TC,W); title('Work done vs Hot end Temperature');
subplot(5,2,2);
plot(TC,COP); title('COP vs Hot end Temperature');
```

**PROGRAM NO.4**

**TITLE: ($P_{max}$, $P_{min}$, Qe, W, COP) v/s α**

```
TE = 80;
TC = 300;
VE = 1*10^(-6);    % 1 cc expansion volume
Pmean = 25*10^(5); % 25 bar
alpha = 60;
x=1;
y=2;
VD = y*VE;
t = TC/TE;
TD = (TE + TC)/2;
S = (VD*TC)/(VE*TD);
i=1;
while(alpha<(131))
    theta = atand((x*sind(alpha))/(t + x*cosd(alpha)));
    A = sqrt( t^2 + 2*t*x*cosd(alpha) + x^2 );
    B = t + x + 2*S;
    delta = A/B;
    Pmax = Pmean*sqrt((1+delta)/(1-delta));
    Pmin = Pmean*sqrt((1-delta)/(1+delta));
    Q = (pi*Pmean*VE*delta*sind(theta))/(1 + sqrt( 1 - delta^2));
    Qc = (pi*Pmean*VE*x*delta*sind(theta-alpha))/(1 + sqrt(1-delta^2));
    w = Q - Qc;
    cop = Q/(Q - Qc);
    Pm1(i)= Pmax;
    Pm2(i)= Pmin;
    Qe(i)= Q;
    W(i)= w;
```

```
    COP(i)= cop;
    i=i+1;
%     disp(i);
    alpha = alpha + 1;
%     disp(y);
end
disp(alpha);
disp(i);
alpha = [60:1:130];
% disp(x);
disp(Qe);
subplot(5,1,4);
plot(alpha,Pm1); title('Pmax vs alpha');
subplot(5,1,2);
plot(alpha,Pm2); title('Pmin vs alpha');
subplot(5,1,3);
plot(alpha,Qe); title('Refrigeration vs alpha');
subplot(5,2,1);
plot(alpha,W); title('Work done vs alpha');
subplot(5,2,2);
plot(alpha,COP); title('COP vs alpha');
```

**PROGRAM NO.5**

**TITLE: ($P_{max}$, $P_{min}$, Qe, W, COP) v/s x**

```
TE = 80;
TC = 300;
VE = 1*10^(-6);    % 1 cc expansion volume
Pmean = 25*10^(5); % 25 bar
alpha = 90;
x=0.8;
y=2;
VD = y*VE;
t = TC/TE;
TD = (TE + TC)/2;
S = (VD*TC)/(VE*TD);
i=1;
while(x<(1.71))
    theta = atand((x*sind(alpha))/(t + x*cosd(alpha)));
    A = sqrt( t^2 + 2*t*x*cosd(alpha) + x^2 );
    B = t + x + 2*S;
    delta = A/B;
    Pmax = Pmean*sqrt((1+delta)/(1-delta));
    Pmin = Pmean*sqrt((1-delta)/(1+delta));
    Q = (pi*Pmean*VE*delta*sind(theta))/(1 + sqrt( 1 - delta^2));
    Qc = (pi*Pmean*VE*x*delta*sind(theta-alpha))/(1 + sqrt(1-delta^2));
    w = Q - Qc;
    cop = Q/(Q - Qc);
    Pm1(i)= Pmax;
    Pm2(i)= Pmin;
    Qe(i)= Q;
    W(i)= w;
    COP(i)= cop;
    i=i+1;
%     disp(i);
```

```
    x=x+0.01;
%     disp(x);
end
% disp(x);
% disp(i);
x = [0.8:0.01:1.7];
% disp(x);
disp(Qe);
subplot(5,1,4);
plot(x,Pm1); title('Pmax vs x');
subplot(5,1,2);
plot(x,Pm2); title('Pmin vs x');
subplot(5,1,3);
plot(x,Qe); title('Refrigeration vs x');
subplot(5,2,1);
plot(x,W); title('Work done vs x');
subplot(5,2,2);
plot(x,COP); title('COP vs x');
```

**PROGRAM NO.6**

**TITLE: ($P_{max}$, $P_{min}$, Qe, W, COP) v/s y**

```
TE = 80;
TC = 300;
VE = 1*10^(-6);    % 1 cc expansion volume
Pmean = 25*10^(5); % 25 bar
alpha = 90;
x=1;
y=1;
t = TC/TE;
theta = atand((x*sind(alpha))/(t + x*cosd(alpha)));
A = sqrt( t^2 + 2*t*x*cosd(alpha) + x^2 );
TD = (TE + TC)/2;
i=1;
while(y<(2.5))
    VD = y*VE;
    S = (VD*TC)/(VE*TD);
    B = t + x + 2*S;
    delta = A/B;
    Pmax = Pmean*sqrt((1+delta)/(1-delta));
    Pmin = Pmean*sqrt((1-delta)/(1+delta));
    Q = (pi*Pmean*VE*delta*sind(theta))/(1 + sqrt( 1 - delta^2));
    Qc = (pi*Pmean*VE*x*delta*sind(theta-alpha))/(1 + sqrt(1-delta^2));
    w = Q - Qc;
    cop = Q/(Q - Qc);
    Pm1(i)= Pmax;
    Pm2(i)= Pmin;
    Qe(i)= Q;
    W(i)= w;
    COP(i)= cop;
    i=i+1;
%     disp(i);
    y=y+0.01;
%     disp(y);
end
```

```
disp(y);
disp(i);
y = [1:0.01:2.5];
% disp(x);
disp(Qe);
subplot(5,1,4);
plot(y,Pm1); title('Pmax vs y');
subplot(5,1,2);
plot(y,Pm2); title('Pmin vs y');
subplot(5,1,3);
plot(y,Qe); title('Refrigeration vs y');
subplot(5,2,1);
plot(y,W); title('Work done vs y');
subplot(5,2,2);
plot(y,COP); title('COP vs y');
```